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Research Article

A mathematical problem-solving framework-based Integrated STEM: Theory and practice

Ijtihadi Kamilia Amalina*[1] & Tibor Vidákovich[2]

[1] Doctoral School of Education, University of Szeged, Hungary, 6722
[2] Institute of Education, University of Szeged, Hungary, 6722
*Corresponding Author: ijtihadi.kamilia.amalina@edu.u-szeged.hu | Phone Number: +36205340488

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ABSTRACT

STEM problem-solving is a core standard in education to tackle the 21st-century challenge. However, the under-represented mathematics in STEM assessment has become an issue. Moreover, the existing frameworks of STEM problem-solving are suitable for assessing activity during teaching and learning. This study covers the limitations by developing a new mathematical problem-solving-based integrated STEM framework and applying it into practice. The new framework combines and integrates frameworks in every STEM discipline. It applies in developing scenario based-test with prompting questions to show the practical use of the framework. The test was administered to 31 of 7th graders students (M=12.29, SD=.46) to prove the consistency of the new framework and the effective role of scenario and prompting questions. The results revealed four types of flows in the problem-solving process posed by students with different in evaluating, representing, and designing phases. The flow of problem-solving and the indicators in a new model were consistent but shown as a dependent process. The scenario and prompting questions worked effectively regardless of a multidimensional prompting question. Several recommendations for developing a test based on the new framework are discussed. Quantitative analysis with large sample size is required for future study.

Keywords: Framework; Mathematics; Problem-Solving; STEM

1. INTRODUCTION

Problem-solving is related to daily life demands and cannot be separated from our life (Erdem-Keklik, 2013; Verschaffel et al., 2020). Hence, it requires proficiency in the curriculum, especially mathematics and science (Erdem-Keklik, 2013; Velasquez & Bueno, 2019). Moreover, some occupations will not appear several five years later because changed by technology. These are the reasons that problem-solving skills, creativity, and critical thinking are essential to tackle 21st-century challenges (Evans et al., 2020; Priemer et al., 2020).

Several researchers proposed problem-solving in the monodisciplinary area (Fan & Zhu, 2000; Gray et al., 2005; Mehalik et al., 2008; Osborne, 2013; Pedaste et al., 2015; Polya, 1978; Tambychik & Meerah, 2010; Windschitl et al., 2008; Donald R. Woods, 2000). It contrasts with the needs of problem-solving in daily life that could not be predicted, ill-defined, and transdisciplinary (Ozturk & Guven, 2016). The concept of STEM problem-solving answers the issue that focuses on integrating two or more subjects related to real-life and 21st- century skills (Kelley & Knowles, 2016; Priemer et al., 2020; Yata et al., 2020). The preliminary framework of mathematical problem-solving is heuristic (Polya, 1978). The importance of problem-solving and a gap among students' achievement in problem-solving initiated the OECD to develop a framework and international assessment in creative problem-solving called the 'Program of International Students Assessment (PISA)' in 2012 related to daily life context (OECD, 2013). Not only international assessment but also several researchers develop mathematical problem-solving frameworks (Tambychik & Meerah, 2010; Wu & Adams, 2006). However, all of them were still focused on the monodisciplinary area and didn't represent the framework's application into practice.

The view of problem-solving is changing from monodisciplinary to interdisciplinary through STEM education. Frameworks of problem-solving were developed that combine engineering design, scientific inquiry, technology literacy or technology design, and mathematical thinking (Kelley & Knowles, 2016; Wells, 2016; Yata et al., 2020). Moreover, Priemer et al. (2020) proposed a framework that applied the PISA framework as a ground theory and embedded some domain-specific problem-solving. However, these frameworks suit measuring problem-solving activity or practical problem-solving (during the teaching and learning process). Moreover, most assessments developed in STEM problem-solving were emphasized science, e.g., Al-Chemist, ChemLabBuilder, Use Your Brainz (Annaggar & Rudiger, 2020; Scherer et al., 2014; Shute et al., 2016). Mathematics as a foundation in applying STEM problem-solving was underrepresented (Maass et al., 2019). Therefore, we need to develop a framework or model of mathematical problem-solving that integrates STEM for a diagnostic test and represents the application into the practice.

The assessment of the cognitive domain in problem-solving is the primary purpose, but the problem-solving in a metacognitive domain is also the vital focus. Metacognition in problem-solving focuses on planning, monitoring, and evaluating (Aurah et al., 2014; Toraman et al., 2020). However, based on the literature review, there is no metacognitive problem-solving in the interdisciplinary area. Therefore, developing a mathematical problem-solving framework-based integrated STEM that focuses on both cognitive and metacognition is required.

According to the needs of a mathematical problem-solving framework integrated STEM theory that combines cognitive and metacognitive domains for diagnostic assessment, then we discuss several questions related to (1) What is a framework of mathematical problem-solving based-integrated STEM theory?; (2) How to apply the framework into the practice by developing a test?; and (3) What is the consistency of the framework that applied into the test?.

2. LITERATURE REVIEW

The framework of problem-solving in PISA is called creative problem-solving: exploring and understanding; representing and formulating; planning and executing; monitoring and reflecting (OECD, 2013).

The other framework about problem-solving, in general, is proposed by Merril et al. (2017). However, the part of representing and formulating was missing in this framework. Moreover, in the problem-solving process, critical thinking is taking part. This reason inspired the researcher to develop a problem-solving framework from FRISCO critical thinking: (1) analyzing the statement or problem; (2) identifying reason of the statement; (3) carrying out the solution; (4) surrounding the problem; (5) checking of the explanation and process (Vargas, 2018).

2.1 Problem-Solving Framework in Mathematics

Mathematics in STEM problem-solving is defined as applying mathematics concepts and procedures (i.e., mathematical thinking) (Jolly, 2016; Kelley & Knowles, 2016). The premier framework of mathematics problem-solving belongs to heuristics problem-solving, namely understanding a problem, devising a plan, carrying out the plan, and looking back (Polya, 1978). This framework is a basis for developing other frameworks in this area and analyzing problem-solving skills. Several researchers create a framework in mathematics problem-solving with the foundation of heuristic problem-solving (Bayazit, 2013; Stylianou, 2011; Tambychik & Meerah, 2010; Wu & Adams, 2006). They emphasized applying mathematics concepts, procedure, rule, fact, mathematization, and standard computational skills in carrying out phase (Bayazit, 2013; Wu & Adams, 2006).

The term "hypothesis" in scientific inquiry is similar to the term "plan" in mathematics heuristic problem-solving (Priemer et al., 2020). Researchers exploit the mathematics approach because it is closely related to scientific inquiry, such as mathematics experimentation and inquiry-based mathematics education. The opposite inquiry concept applied in mathematical problem-solving is proving. It uses a deductive approach to generate a conclusion that does not necessarily require empirical data (Boero, 1999; Priemer et al., 2020).

Problem-solving in mathematics itself has been measured in PISA with a term of literacy. It allows students to employ mathematics facts, procedures, concepts, and reasoning in various contexts related to daily life (OECD, 2013). The OECD developed a framework for mathematics PISA 2012 and renewed the need for 21st-century skills and balance among mathematical reasoning, problem-solving, mathematical content, and context. The item formats are constructed and selected responses in several contexts, including personal, occupational, societal, and scientific. It has several knowledge contents related to change & relationship, space & shape, quantity, and uncertainty & data (OECD, 2018).

Metacognitive skills are implied and practiced during mathematical problem-solving (Bakar & Ismail, 2020; Velasquez & Bueno, 2019). Metacognitive in problem-solving focuses on planning, monitoring, evaluating, and reflection (Aurah et al., 2014; Velasquez & Bueno, 2019). Schraw & Dennison (1994) divided metacognitive awareness into two major components: knowledge about cognition and regulation of cognition. Knowledge about cognition facilitates reflective aspects, including declarative knowledge (i.e., knowledge about strategy), procedural knowledge (i.e., knowledge about how to use strategy), and conditional knowledge (knowledge about when and why to use strategy). Regulation of cognition facilitates the control aspect. Table 1 explains the problem-solving phase and the metacognitive skills involved in them.

Table 1. Metacognitive Skills in Problem-Solving

Problem-Solving Phases	Metacognitive skills involved them
Understanding a problem	Declarative, procedural, planning
Devising a plan	Conditional, monitoring, planning
Carrying out the plan	Planning, monitoring, evaluating
Looking back	Monitoring, evaluating

2.2 Problem-Solving Framework in Science

Science in STEM education defines the application of science concepts (Jolly, 2016). However, it could refer to scientific inquiry (Pedaste et al., 2015; Rudibyani et al., 2020; Windschitl et al., 2008) and scientific reasoning (Osborne, 2013). Pedaste et al. (2015) indicated a cyclic model for scientific inquiry: orientation, questioning, exploration, data interpretation, and conclusion. Moreover, a linear model of scientific inquiry includes organizing the information that is already known or would like to know, generating hypotheses, seeking evidence, constructing an argument, setting broad parameters, and metacognitive prompts. The phases of scientific reasoning are similar to scientific inquiry, including investigating problems, observing, experimenting, measuring, testing, generating hypotheses, developing explanations and solutions, and evaluating.

Several well-known frameworks and frameworks proposed by researchers are contributing to science problem-solving. The Minnesota Assessment of Problem Solving (MAPS) framework is a well-known framework in physics problem-solving (Docktor et al., 2016). It includes some steps focused on sciences: (1) Visualization/problem description; (2) physics approach; (3) specific application of physics concept; (4) application of mathematical procedure; and (5) logical conclusion. Another well-known assessment is the international assessment in science in the PISA, using science literacy (OECD, 2013). The contexts are related to personal and global (health, natural resource, environment, hazard, and frontiers of science and technology). A specific framework in physics education determines the equations that are suitable for solving problems, substituting known values for equations, and evaluating solutions (Naqiyah et al., 2020).

2.3 Problem-Solving Framework in Engineering and Technology

This part will be described several problem-solving frameworks in engineering and technology. Even though technology and engineering are not the same, there are no profound differences concerning problem-solving activities in education. The engineering area includes STEM assessment by focusing on engineering design or creating a solution (Jolly, 2016; Kelley & Knowles, 2016; Wells, 2016). Engineering design allows connecting science and mathematical concepts (Shahali et al., 2017; Shanta & Wells, 2020). When engineering is a process of applying mathematics and science, then technology is a result (Jolly, 2016; Priemer et al., 2020). The technology could also be defined as physical tools or artifacts that ease human works (Jolly, 2016; Kelley & Knowles, 2016).

Problem-solving in engineering could be defined as a design process concept (Baele, 2017; Kelley & Knowles, 2016; Wells, 2016; Yata et al., 2020). Baele (2017) indicated an engineering design: (1) Asking; (2) Imagining; (3) Planning; (4) Creating; (5) Testing; and (6) Evaluating. Another researcher proposed (1) identify need/problem; (2) research need/problem; (3) develop possible solution; (4) select best possible solution; (5) construct prototype; (6) test and evaluate solution; (7) communicate solution; (8) redesign; and (9) finalization design (Robinson, 2016).

The concept of engineering problem solving similar to heuristic problem-solving suggested a linear model, namely roadmap, modeling, governing equations, computation, discussion, and verification (e.g., Gray et al., 2005). They supported the study by providing essay problems (statics and dynamics problems). Furthermore, Wood (2000) proposed a cyclic model called: engage; define a stated problem; explore; plan a solution; do it (carry out the plan); evaluate, check & look back. These models are similar to scientific inquiry problem-solving in certain phases (e.g., constituting a question, developing an empirical test, and making a conclusion).

2.4 Framework in Practical Problem-Solving

The Priposal model is a cyclic conceptual/pedagogical framework model of integrative STEM education. It is constructed from engineering and technology design (Wells, 2016). It consists of problem identification (need, define, formulate), ideation (criteria, brainstorm, generate), research (explore, investigate, examine), potential solutions (analyze, visualize, select), optimization (experiment, revisit, construct), solution evaluation (test, analyze, interpret), alterations (identify, redesign, retest), and learned outcomes (process, iterations, justify).

Kelley & Knowles (2016) proposed a STEM education framework for secondary school or high school level. It combines four domains: engineering design, scientific inquiry, technology literacy, mathematical thinking, and situated STEM learning in an activity. Engineering design and scientific inquiry are integrators, while situated STEM learning is a physical and social condition before learning. These domains are not necessarily occur during every STEM activity. Yata et al. (2020) conducted a similar framework that exploits engineering design as an integrator to connect science, mathematics, and technology concepts.

Another framework was developed by utilizing a PISA creative problem-solving framework as a ground problem-solving framework and embedded some domain-specific problem solving, such as scientific inquiry, proving, engineering design, and computer science (Priemer et al., 2020).

3. RESEARCH METHOD

The study aims to develop a new framework in mathematical problem-solving-based-integrated STEM and apply it into practice to prove its consistency. There are four steps for developing a new framework of mathematical problem-solving-based integrated STEM theory. The first step is to choose the general structure or framework to be the main framework of the problem-solving. The second step is to go through all domain-specific frameworks and assign problem-solving indicators to the general structure. The third step is to eliminate duplicate and synonym problem-solving indicators in the domain-specific frameworks, and the fourth step is to order the indicators into a flow chart and show it as a process. After developing a framework of mathematical problem-solving integrated STEM theory, we created a test to show the application of the framework. The test was then administered to 31 7th grader students, 12 boys and 19 girls (M=12.29, SD=.46) from a private school in Surabaya, Indonesia. It aims to show the consistency of problem-solving indicators and flow from the theory to practice. It is also aimed to prove the framework's suitability with the test. The data of 31 students will be analyzed using descriptive statistics to represent students' percentages in every different problem-solving process (flow). Several different students' answers are provided to describe the emergence of problem-solving indicators and flows (process) shown by students.

4. RESULTS AND DISCUSSION

4.1 A new framework of mathematical problem-solving integrated STEM

Based on the literature review, the definition of a mathematical problem integrated STEM is a problem that applies mathematics contents and concepts in the context of science by using a design-based process with technology. It is similar to the mathematics word problem related to science phenomena and using design-based and technology to solve it. Mathematics takes part in the contents and concepts used, and science participates in a context and the way of thinking. Engineering partakes in the design-based process, and technology is the tools (both electronic and non-electronic).

From several general frameworks in problem-solving, the PISA creative problem-solving framework could cover other frameworks, e.g., a framework from Merril et al. (2017) and Vargas (2018). The PISA framework also can apply in multi-disciplines. Therefore, the PISA framework could be applied as a ground framework to develop a new framework that accommodates critical thinking. To generate indicators or steps in each PISA framework, we develop from each discipline by combining them and eliminating the duplicate models. Several theories in mathematics problem-solving will be used. Heuristic problem-solving, mathematics experimentation and inquiry-based mathematics are applied. Therefore, making a hypothesis is also part of mathematics problem-solving. Also, proving theory involves making a hypothesis, selecting a reasonable argument, and deducting, proofing, or finding a counterexample. One of the Wu & Adam (2006) framework steps inspires to development of indicator 8 in the new model. Hence, mathematics problem-solving indicators are 1, 2, 3, 4, 5, 6, 7, 8, and 9.

The framework of science problem-solving that covers all other frameworks is scientific inquiry. It emphasizes understanding, generating hypotheses, and seeking and developing reasonable arguments. Therefore, science problem-solving indicators are 1, 2, 3, 4, 5, 7, 8, and 9. The framework of engineering and technology employed in this study is engineering-design based. It emphasized creating (designing), building models, and testing solutions. Because the framework will be conducted for non-practical problem-solving, testing the solution defines as evaluating and ensuring whether the answer is correct. The steps representing engineering & technology problem-solving are 1, 2, 5, 6, 7, 8, and 9. Technology will be emphasized in applying the tools during a problem-solving task (e.g., calculator, ruler, grid paper etc.) Metacognitive skills appear in the indicators 1 (declarative), 2, 4 (procedural), 6 (conceptual), 3, 5 (planning), 7, 8, 9 (monitoring and evaluating). It is also will be included metacognitive prompts in the monitoring and evaluating process. The new framework of STEM problem solving is described in Table 2.

Table 2. Indicators of Mathematical Problem-Solving Framework-based Integrated STEM

	Steps
Exploring & understanding	1. Determining the goal of a problem (STEM); 2. Identifying the relevant and valuable unknown and given information (STEM); 3. Utilizing the basic concept that can be useful (SM)
Representing & formulating	4. Representing the problem by constructing tabular, graphical, symbolic, or verbal representations using technology (STM); 5. Making a prediction (hypothesis), developing criteria, or checking existing theory (STEM).
Planning & executing	6. Developing the reasonable argument, explanation, and solution (including strategy, design the steps, and building the model) (TEM); 7. Arranging, critically choosing, and evaluating alternatives (STEM); 8. Devising a plan by deducting, proofing, finding a counterexample, applying mathematics concepts, mathematization, reasoning, computational skills, science concept, and technology (STEM).
Monitoring & reflecting	9. Making conclusions, evaluating, and reflecting on results and methods (STEM).

The nine problem-solving indicators are ordered into the flowchart to show them as a process. The evaluation, reflection, and conclusion indicators can prevail after every problem-solving indicator. It is because before deciding, we need to evaluate to address a conclusion. An indicator related to representing a problem can occur parallel with the step in identifying given and unknown information. Figure 1 illustrates the flow of problem-solving indicators.

4.2 Applying the Framework into a Test Development: Theory to Practice

The new framework implemented design-based engineering as an integrator. Hence, it is necessary to think about the idea of a test that contains a design activity. However, it is not an obligation to apply the framework to the design activity test. The framework could still be used for a test without design activity because it can be changed into a designing strategy. However, the test should require students to apply several strategies or models. This type of test will lead to the use of technology. Students will use several tools when the test requires a design activity (e.g., ruler, crayon, grid paper, etc.).

Figure 1. Visual Representation of the Mathematical Problem-Solving Framework-based Integrated STEM

Scenario

Eco-friendly and low budget product is starting to be prioritized. We want to produce a packaging with the criteria:

1. The packaging has length, width, and height of 18cm x 18cm x 8cm respectively
2. The cover of the packaging is square with 1.2 times the area of the base.
3. There are four material options:

	Duplex paper	Ivory paper	Styrofoam	Plastic
Size	79×109cm	215× 330*mm*	100 × 50cm	17×50cm
Thickness	250gsm	250gsm	0.5cm	0.5cm
Price	Rp 4,000.00	Rp 2,000.00	Rp 9,000.00	Rp 1,300.00
The time needed to decompose	2-6 months		Can't decompose	1000 years

Challenge

Design an eco-friendly packaging and decide the lowest budget of a packaging production based on these criteria

Prompting Questions:

1. What is your challenge (asked) in the scenario?
2. What information do you need to answer the challenge?
3. What is the total area of 4 sides, cover, and base in cm^2?
4. What is your guess related to the type of material that could be used to design the packaging? Give your reasons
5. Draw your packaging design based on the shape and lengths
6. Mention two materials that are impossible to be chosen and give the reasons!
7. How much is the price for your packaging based on your chosen material and the area of your packaging?
8. Is the packaging eco-friendly and the cheapest one? Give your conclusion about the price and type of material used to produce packaging.

Figure 2. Example of test developed based on a new framework of mathematical problem-solving based integrated STEM

The indicators of making a hypothesis, arranging, and evaluating alternatives have required a test with several options or choices. The best item to represent these indicators is the item that requests students to decide the most effective option from several given options. The indicator "representing the problem" is not always can be assessed directly. However, we could examine the statement or students' answers during the "exploring and understanding" indicators. We highlight several essential things that need to be considered: (1) search a scientific phenomenon that can be applied as a context; (2) consider curriculum requirements in mathematics that can be applied as the contents if the test is curriculum-oriented; (3) create a scenario that provides student to explore their design skill; (4) give several options as a challenge to assess their evaluation and making hypothesis skills. The most appropriate mathematics contents that can be integrated into the STEM are statistics (data, representation, and central tendency), geometry, number, and measurement(Lasa et al., 2020; Maass et al., 2019). We provide an example that tries to bring out all the indicators of a new framework in Figure 2. The test is a scenario-based test with several prompting questions to explore every indicator of mathematical problem-solving framework-based integrated STEM. The prompting question also aims to reveal students' thinking to make sure students' answers are consistent and address student ability identified.

The test example provides an opportunity for students to design, develop a hypothesis, and evaluate their choice. However, the "representing the problem" indicator is not visible directly by this item. The prompting question 1 and 2 facilitate students to understand the problem, given or unknown information related to the problem, and the aim of the problem. The indicator of understanding the given and unknown information can be assessed by how students represent the problem into a mathematical symbol. Prompting question 3 explores the basic concepts to be used for solving the problem. Prompting question 4 is for analyzing how they make a hypothesis with a reasonable argument while prompting question 5 is assessing how they develop or design a model, and prompting question 6 is evaluating alternatives. Prompting question 7 gives students a chance to explore their mathematics knowledge, and prompting question 8 evaluates and gives a conclusion.

4.3 The Result of Students Answer in the Developed Test

The results of students' answers are analyzed to prove the consistency of the problem-solving process (flow) and the suitability of the framework and developed test. There were four different processes or flows regarding how students answer the test. However, some common errors and mistakes were still encountered: (1) errors in writing the results, although, in the previous step of problem-solving, students wrote the correct result; and (2) errors in selecting information that impacted the calculation. Figure 3 illustrates the different flow charts of students' problem-solving.

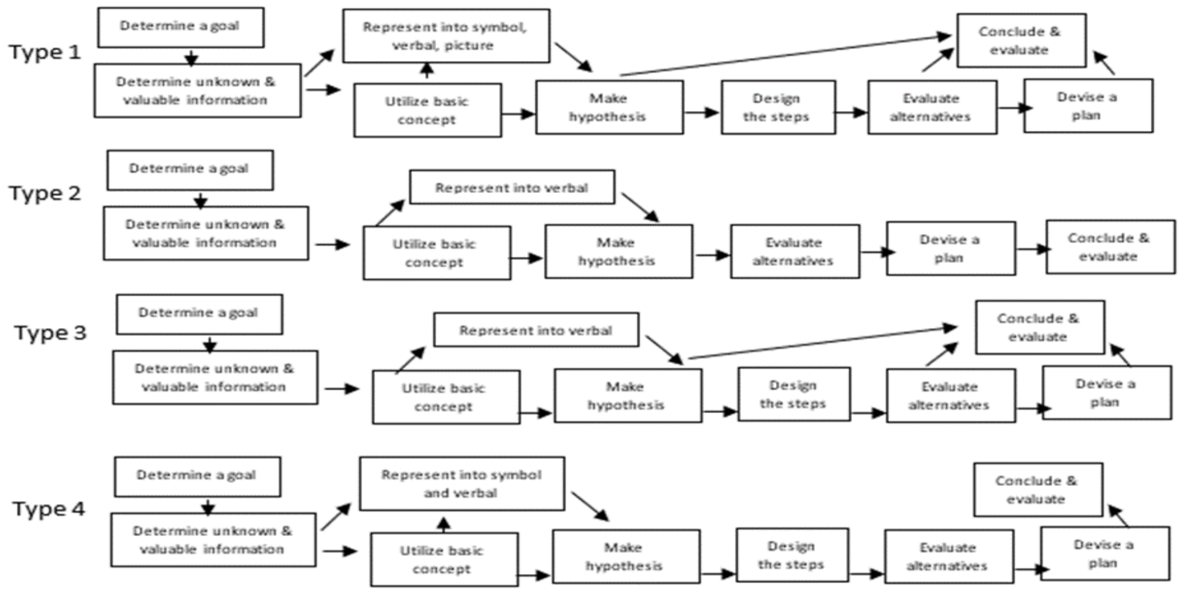

Figure 3. Different flowchart of problem-solving step

The differences among these flow charts are in the representation, design, and evaluation indicators. Nine students (29%) showed the type 3 flow chart in the problem-solving process. They did not represent the problem in symbolic but verbal representation. Both representations are acceptable since the problem does not require mathematical modeling to express it. Three students (9.7%) were in the type 2 flow chart (did not indicate the design process). The students were able to move into the next indicators. However, the answers were incorrect. Hence, the design process is necessary for the test. 24 out of 11 students (77.4%) did not show the evaluation indicator implicitly during their problem-solving process. However, they did the logical steps of problem-solving. It means that the evaluation phase is not always can be assessed explicitly. Moreover, the evaluation process could be evaluated by using a metacognitive prompting question, e.g., is your choice appropriate with the given criteria?". The metacognitive prompting question could not assess the process of how students evaluate their steps during problem-solving. It only considers the product of the evaluation process. Hence, more than one assessment is needed, e.g., interview or observation.

The consistency of the problem-solving indicators and their flow in the new framework can be proved in a practice area. However, the prompting question is required to stimulate and hint students to apply their prior knowledge, experience, and understanding. The prompting question also keeps a problem-solving process on track. To show that the prompting questions and the scenario work effectively, we analyze every step of students' answers and provide three different examples from students A, B, and C in Figures 4, 5, and 6, respectively.

Student A's answer represented that an evaluation process occurs explicitly through the attached calculation (or representation), e.g., in prompting question's answer number 3 and number 4, there is a picture and calculation for helping and evaluating. The explicit evaluation phase makes student A's answer different from other students. We highlighted the student's answer in prompting question 2. The prompting question stimulated student A to determine not only the given information but also the unknown information. Moreover, the prompting question in number 2 could also hint at the representation phase. Hence, this prompting question is a multi-dimension question. Student A's answer in numbers 4-8 confirmed that the target of prompting questions are achieved. Therefore, all the prompting questions work effectively and guide students to the track of problem-solving based on student a answer.

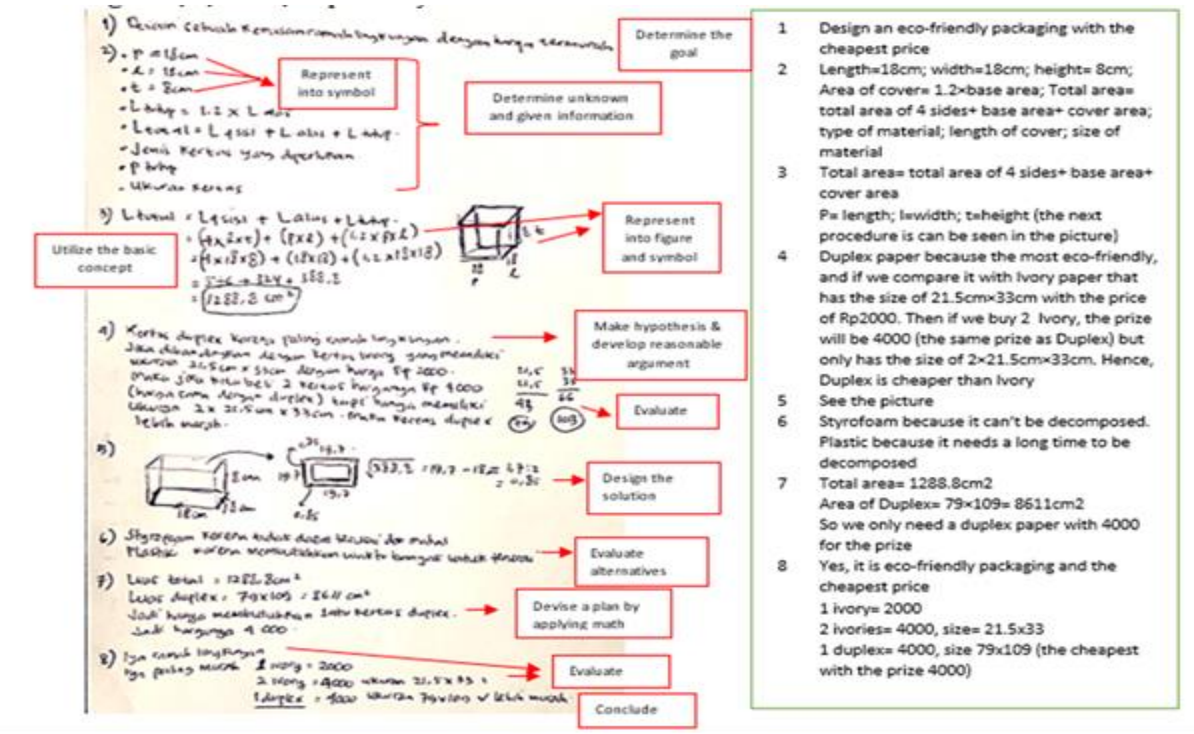

Figure 4. An example of student A's answer (flow chart type 1)

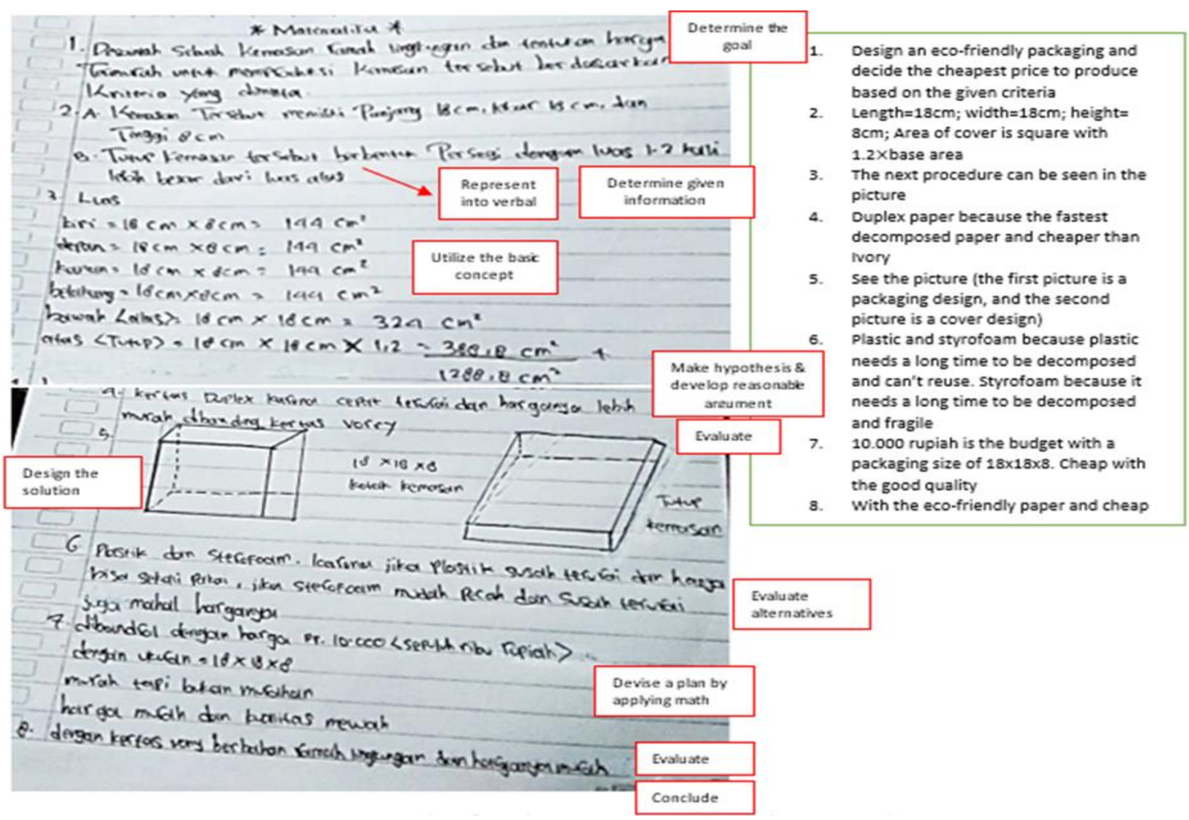

Figure 5. An example of student B's answer (flow chart type 3)

Several issues are highlighted in student B's answer. First, prompting question number 2 did not work effectively because student B only determined the given information. However, the answer of number 3 showed that student B understood the important unknown information for answering the challenge. Hence, more specific prompting question 2 is required (e.g., what is the given and unknown information needed to answer the challenge?). Second, the different representation was detected in number 2, verbal representation. It is acceptable since different types of representation can occur in a problem. Third, the evaluation phase in the "make a hypothesis, and reasonable argument" indicators were revealed implicitly. The best way to assess the evaluation phase is to consider the whole problem-solving process. Hence, the assessor could decide the quality of students' evaluation skills.

The problem-solving indicators will be distracted if students fail in a specific phase (indicator). Student B did well in the initial problem-solving indicator to the "evaluate alternative" indicators. However, student B failed in devising a plan, and it affected the conclusion. It can be proved that the problem-solving steps or indicators packed in the flowchart are dependent on each other. Another type of problem-solving process was showed by student C. The answer determines that student C understood the scenario, the challenge, and how to solve the challenge (see Figure 6). However, there were several errors in analyzing the given information and calculating the basic concept, e.g., the size of ivory paper is in mm, but it is written cm in student C's answer. It affected other steps of problem-solving. The error is not causing the changing flow (process) of problem-solving based on the new framework. The flow showed by student C was consistent with the developed new framework. All the prompting questions worked effectively. However, the evaluation phase did not appear in student C's answer, which influenced her/him to make an error. Hence, it concludes that error in solving a problem will not change the flow of problem-solving but it will effect the quality of students' answer in the next problem-solving step (indicators).

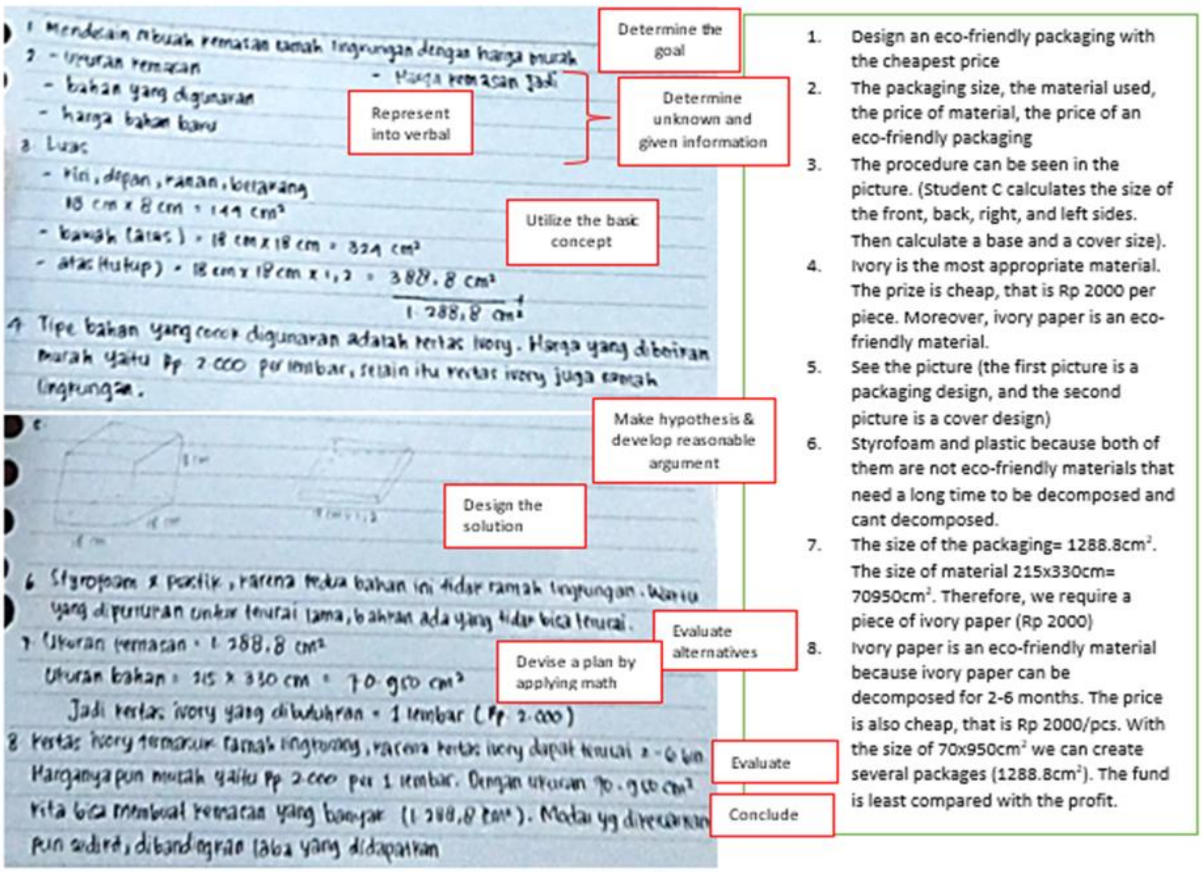

Figure 6. An example of student C's answer (flow chart type 4)

5. CONCLUSION

A new framework of mathematical problem-solving-based-integrated STEM is developed to answer the limitation of frameworks in non-practical problem-solving. The framework is developed from a PISA creative problem-solving as a ground framework and generated indicators from several frameworks in each discipline (i.e., heuristic, mathematics experimentation, proving, inquiry, and engineering design-based). A new framework can be applied to develop a test integrating mathematics with STEM theory. The scenario-based problem test with prompting questions is appropriate for measuring mathematical problem-solving-based integrated STEM. Hence, creating a scenario that facilitates students to explore their design skills in a scientific phenomenon using mathematics contents is required. The flow and indicators of problem-solving are valid according to the students' answers. The flow and indicators will not be distracted by student errors in calculation or understanding information. These errors will impact their quality of answer (right/ wrong) in the next steps. Hence, it is proved that problem-solving steps or phases are shown as a dependent process but fixed; one influences the other. However, students' probability of different problem representations is acceptable and wide. It depends on their strategy to interpret the problem and information. The evaluation phase is difficult to assess explicitly. The solution is using a metacognitive prompting question. However, it can only assess the product rather than the process of students' evaluation. The role of different assessment types is required in this case. Analyzing students' whole answers could be considered to assess the evaluation process. However, we could not build it as a prompting question. Prompting questions in the developed problem-solving framework are necessary to stimulate students' prior knowledge and experience. Moreover, it can be designed to make students always on track in a problem-solving flow, give them a hint, and reveal students' thinking. However, the clear and specific prompting question is important to avoid students' misunderstanding. The problem of multidimensional prompting items could not be separated from problem-solving since problem-solving is seen as a process rather than only a result itself.

RECOMMENDATIONS

STEM problem-solving is a core standard in education to tackle the 21st-century challenge. However, the under-represented mathematics in STEM assessment has become an issue. Moreover, the existing frameworks of STEM problem-solving are suitable for assessing activity during teaching and learning. This study covers the limitations by developing a new mathematical problem-solving-based integrated STEM framework and applying it into practice. The new framework combines and integrates frameworks in every STEM discipline. It applies in developing scenario based-test with prompting questions to show the practical use of the framework. The results revealed four types of flows in the problem-solving process posed by students with different in evaluating, representing, and designing phases. The flow of problem-solving and the indicators in a new model were consistent but shown as a dependent process. The scenario and prompting questions worked effectively regardless of a multidimensional prompting question. Several recommendations for developing a test based on the new framework are discussed. Quantitative analysis with large sample size is required for future study.

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AUTHOR'S CONTRIBUTIONS

All authors discussed the results and contributed to from the start to final manuscript.

CONFLICT OF INTEREST

There are no conflicts of interest declared by the authors.

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