

Research Article

An exploration of grade 2 learners' experiences in solving addition and subtraction word problems

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ABSTRACT

Mathematics has always been a major part of school curricula in most countries. It provides a variety of useful tools that help students to solve problems that they encounter in everyday life. Mathematical tools are considered powerful, as long as one knows how to employ them across a range of suitable situations. However, selecting and using these tools appropriately appear to be challenging for many students. Because of this, word problems have been included in mathematics education to offer practice for students in applying mathematical skills effectively in various problem situations confronted in everyday circumstances. In addition, there is limited research about how learners in the foundation phase (Grades 1-3) solve addition and subtraction word problems. A relatively recent review of South African studies on mathematics education has revealed a paucity of research at the primary school level. This study therefore seeks to explore grade 2 learners' experiences in solving addition and subtraction word problems. A qualitative research was conducted to describe grade 2 learners' experience whilst solving addition and subtraction problems. Only 6 learners in grade 2 were selected to participate in this study. Learners demonstrated a strong preference for paper and pencil methods as well as the standard algorithm, when solving problems in word format. However, little evidence exists within the study that any of the learners conceptually understood the algorithm. The use of base 10 blocks to model the partial sums and partial differences algorithms would help most of the learners to develop a conceptual understanding of addition and subtraction of word problems in mathematics.

Keywords: Addition and Subtraction; Cognition-Based Assessment (CBA); Standard Algorithm; Word Problems;

1. INTRODUCTION

Addition and subtraction have long been topics of study from early childhood development (ECD) through grade 2 of primary school (Carpenter et al., 2020). Research examined how learners in primary school work with single-digit numbers and sums of 20 or less. Research on cognitively guided instruction (CGI) (Carpenter et al., 2020), suggests that learners are able to solve more complex problems than teachers might expect. Learners in elementary grades were able to solve many different types of problems, including multiplication and division problems at early ages. Most primary school programmes make assumptions about addition and subtraction, especially word problems. For example, teachers assume that word problems are best introduced through physical or pictorial representations of putting together or breaking up sets of objects. Another common assumption is that word problems are difficult for learners of all ages, and that learners must master addition and subtraction operations before they can solve even simple word problems. These assumptions could impact on learners' approaches to word problems. In their study, Carpenter et al. (2020) indicate that both assumptions may be false. There is considerable research literature that demonstrates that school learners, especially in the primary school, solve addition and subtraction computation exercises by using several basic counting strategies (Hopkins et al., 2022). This finding identifies the strong relation between types of word problems and solution strategies. However, because a variety of semantically different word problems can be solved by addition or subtraction, the choice of solution strategy becomes more complex. Understanding solution strategies that learners use in solving word problems is at the heart of our understanding of how learners solve addition and subtraction word problems.

Statement of the Problems

A relatively recent review of South African studies on mathematics education has revealed a paucity of research issues at

the primary school level (Venkat, & Askew, 2018). While this review focuses on limited mathematics education research concerns at primary school level, this is compounded by further limitations of studies that focus on the foundation phase (Grades R-3). A broad range of literature has focused on educator and classroom instruction.

In addition, local and international studies have established that South African learners perform poorly in mathematics. According to the Capetown News (2018), there was a downward rate of 54% in 2020 from 58% in 2018 of the students who achieved a mark of at least 30% in mathematics. The 2001 SACMEQ dataset indicates that half the grade 6 mathematics learners perform at grade 3 level or lower (Schollar, 2008). The 2005 Department of Education's systemic evaluation mathematics scores reveals that only one in ten learners were at the required standard by the National Curriculum Statements (NCS) (Fleisch, 2008). This has led to some commentators to conclude that 80% of South African learners perform below the minimum expected standards for their grade (Schollar, 2008). Without a deep understanding of the problem-solving strategies that learners use to solve word problems, it is likely that performance in mathematics will continue to be on a downward spiral. Based on these arguments, the purpose of this study is to explore grade 2 learners' experiences in solving addition and subtraction word problems. The study thus seeks to:

- 1) Examine the strategies that South African grade 2 learners use in solving addition and subtraction word problems.
- 2) Analyse the difficulties grade 2 learners encounter when solving addition and subtraction word problems and how these problems can be abated.

2. LITERATURE REVIEW

An extensive body of assessment data points to poor performance in mathematics across all levels of the school system in South Africa. This data ranges from classroom observation and localised small-scale studies (Ensor et al., 2002; Schollar, 2008) to national representative assessments, such as the Southern and Eastern Consortium for Monitoring Educational Quality (SACMEQ), the Trends in International Mathematics and Science Study (TIMSS) and the National School Effectiveness (NSE). Evidence from national, regional, and international assessments points to the fact that South African learners have a poor grasp of primary foundational concepts (Taylor, 2011). Increasingly these challenges seen most vividly at the endpoint of the system, with high drop-out and failure rates in national grade 12 mathematics assessments, are thought to emanate from the primary school level. This conclusion stems from the recognition that the early stages of learning mathematics impact on what is possible to learn in higher grades. With the introduction of Annual National Assessments (ANAs) in grades 1 to 6 in 2011, more standardised and comparative-assessment information is now available in South Africa on what primary learners are unable to do in mathematics assessments (DBE, 2013). The Foundation Phase ANA diagnostic reports for 2012, 2013 and 2014 all identify addition and subtraction as a recurrent area of weakness (DBE, 2012; DBE, 2013; DBE, 2014) in mathematics.

Conceptual Analysis of Word Problems

Word problems in mathematics are often used in school mathematics to offer learners the opportunity to explore mathematical relationships and structure (Lee & Hwang, 2022). Exploration of mathematical structures and relationships shows that word problems can be connected to problem solving in school mathematics curricula. However, previous research studies have reported that word problems are misused in ways that defeat the original intent of exploring mathematical structure and relationship. Research recommends that learners should be engaged in a variety of truly problematic tasks so that mathematical sense-making is practised (Marcus & Fey, 2003). Carefully chosen word problems can provide a rich context for learning addition and subtraction concepts (Singh & Hoon, 2010). In a comprehensive longitudinal study, Carpenter et al. (2020) were able to show that learners were able to solve many different types of problems, including multiplication and division problems at early ages. This finding contrasts with the South African educational literature, which shows that word problems are problematic, especially for the grade 2 learners.

In another relevant research, Daroczy et al. (2015) aver that word problems are the most difficult and complex topics encountered by learners during their elementary-level mathematical development. Normally in the classroom setting, word problems are regarded as merely arithmetic tasks yet extant literature revealed several linguistic verbal aspects not directly linked to arithmetic, contributing to the learners' difficulties in solving word problems (Daroczy et al., 2015). Story problems require two phases of solving, namely a comprehension phase and a solution phase. During the comprehension phase, the problem solver works to understand the context of the problem and determine a way to represent the problem in order to solve it. During the solution phase, the problem solver uses mathematics and mathematical

symbols to find a solution. As they work to understand and solve the problem, a problem solver might move back and forth between the comprehension phase and the solution phase rather than working sequentially from one phase to another. In their study, Anwar and Rahmawati (2017) found that student success in solving word problems is not out of the representation role. Mathematical problem-solving using representation is thus strongly recommended because it can be a significant tool to facilitate word problem solving process. They also noticed that, while learners had to translate from word format into another format to solve the problem, the learners used informal strategies to solve the problems. Informal strategies were defined as strategies not usually taught during classroom instruction. The study found that many learners used guess-and-test strategies or an unwind strategy to solve word problems. The unwind strategy has the learner work backward through the problem, using inverse operations to find the solution. The researchers noted that, while the guess-and-test strategy was not efficient, the unwind strategy sometimes resulted in less writing than using equations in symbolic form. Anwar and Rahmawati found that when learners solve story problems or word equations, they are able to draw on prior knowledge and intuitive strategies rather than relying on the formal solution process used to solve symbol equations. The researchers encouraged the use of story problems to ground new learning of how to manipulate symbol equations during classroom instruction.

Impact of Language in Solving Word Problems in Mathematics

The low achievement in mathematics takes place within a context of primary schooling which may be labelled challenging (in comparison to prior work undertaken in England and South African private schools). The majority of South African primary schools operate with limited resources, servicing communities living in poverty, where English is not the main language of learners, and where the school culture is frequently not supportive of learning (Carnoy, Chisholm & Chilisa, 2012). This broad context of primary schooling in South Africa was sketched in a recent review of classroom-based studies on South African primary schools.

Research conducted in the South African context has pointed to the challenges of using a language that is not easily accessible to learners and even educators in some cases, while the use of first language in mathematics classrooms has been shown to foster more interactions between learners and educators (Sepeng, 2014). Hoadley (2012) documents that most learners in South African primary schools learn in an additional language and views this factor as compounding the lack of sense-making in mathematics. This is particularly evident in poor attainment in word problems and, as such, some detail on this multi-lingual classroom context and its implications for mathematics teaching and learning is appropriate. In South Africa, from grade 4 upwards the majority of learners learn mathematics in English, which is not their main language. The Department of Education (DOE, 1997) official policy since 1997 dictates that, in the first four years of primary schooling, the language of learning and teaching (LoLT) for all learning areas (including mathematics) is to be in the home language of the learners. The acquisition and development of children's mathematical knowledge in the early stages of learning is based on the language used as the medium of instruction (Edmonds-Wathen *et al.*, 2016). Similarly, Vukovic and Lesaux (2013) aver that language ability is essential for children's mathematical development and that learners' mathematical difficulties may reflect deficient linguistic process.

This implies that many indigenous learners have difficulties when learning mathematics and that the language background of these learners can impact significantly on all educational outcomes (Makamure, 2019). Furthermore, Vukovic and Lesaux (2013) aver that the abstract symbols inherent in mathematics can be conceptualised by learners through language proficiency. It is therefore through language used for instruction that mathematical ideas are conceptualised. In practice, the multiplicity of home languages in urban settings, combined with a 'press' for English due to its being seen as the language associated with socio-economic status, results in English being the language of teaching in many schools (Dalvit, Murray & Terzoli, 2009). The complexity of learning mathematics in English, when this is not the main language of the learners, impacts on all levels of the system (Makamure, 2019). This contextual reality makes it difficult to separate the origin of poor attainment in solving addition and subtraction word problems. The extent to which these difficulties arise either from 'mathematics' or from 'language' or from 'both language and mathematics', is in question. Schollar (2008) opines that the generally poor English reading levels brought into question whether learners' difficulties in solving mathematical word problems in assessments were language-based, or conceptual problems with language and mathematics were most acutely seen when considering learner performance in 'word problems' as opposed to 'bare calculations'.

In support of the concern regarding the connection between mathematics and language learning, Ensor et al. (2002) note that a mathematics assessment (even as high as Grade 7 level), "... cannot be seen in any simple way as a test of mathematical competence. Rather, it is a means of assessing learner's ability to answer mathematics questions, expressed in particular ways in English" (Ensor et al., 2002, p. 30). In another related study, Sepeng (2014) found that language difficulties were evident in both languages, that is, 'computation errors ... seem to stem from the inability to use language(s) (home and/or language of learning and teaching) effectively in order to solve problems in realistic settings. In addition, Sepeng (2014) views the home language and the language of learning and teaching (LoLT) as having complementary roles, in supporting meaning making when solving additive relations. This notion of complementary roles for the home language and the LoLT aligns with that of a learners' main language being viewed as a resource rather than a problem (Adler 2002, p.3). Setati et al. (2002) point out that learning mathematics in multilingual classrooms is not simply a matter of managing the interaction between the LoLT and the main language of a learner. They argue that focusing on the meaning of individual words or the addition of new vocabulary is not adequate, as part of learning mathematics is acquiring control over the mathematics register (p.9). As a result, communicating mathematically in multilingual classrooms means managing, not only the interaction between learners' main language and the LoLT, but also the interactions between ordinary English and mathematical English, between formal and informal mathematics language, and between procedural and conceptual discourses (Setati et al., 2002). It is important to bear in mind that these language issues are not unique to the South African multilingual classroom environment, as there is international documentation on the difficulties with flexibly moving between the semiotic systems of natural language and mathematical numbers for all learners.

Lack of Fluencies and Negative Mathematical Identities

There is evidence of learners having negative mathematical identities within "a passive, overly educator dependent culture of learning mathematics" (Graven, 2012, p.60). Graven (2012) notes that learners' "current motivations for mathematical participation are seemingly dominated by compliance with educator instructions and getting answers right" (p.60). The above empirical findings from the South African Foundation Phase classrooms point to a passive educator-dependent culture. This is compounded by educators not establishing and building on a base of known and established number facts and properties to support learning progression, which is over and above the concerns related to mathematical meaning-making in the challenging schooling context. This lack of sense-making relates to a lack of a 'productive disposition' towards mathematics, which is defined by Kilpatrick et al. (2001) as, "... the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, and to believe that steady effort in mathematics pays off and to see oneself as an effective learner and doer of mathematics" (p.131). Mathematics word problems may, therefore, be eligible to propel sense in learning mathematics.

3. RESEARCH METHOD

A qualitative research was conducted to describe grade 2 learners' experience whilst solving addition and subtraction problems. Only 6 learners in grade 2 were selected to participate in this study. The Annual National Assessment (ANA) diagnostic report identified weakness in solving word problems as a challenging factor to grade 2 learners (DEE, 2014), hence, children aged between the ages of 8 and 9 years were included in the study. All of them were interviewed and observations made on how they solved word problems. Six questions were asked as standard interview questions and three additional extension questions that included 4 or more digit numbers. Learners were asked to pay close attention to how they solve the word problems rather than the answer.

Establishing problem-solving strategies was the motivation for this study that explored ways in which learners use various strategies to solve word problems. The research pursued a theoretical and methodological orientation appropriate to the study of problem-solving phenomena in natural settings. This is because the study had an interest in how grade 2 learners solve addition and subtraction word problems and regarded the associated experiences as naturally occurring instances in problem-solving. Within problem-solving strategies, this study looked at three distinct problem types: result unknown, change unknown and start unknown. In a result unknown problem, the two numbers that are to be operated upon are known and the solver works to find the result of the computation. In a change unknown problem, the initial addend or minuend in the computation is known and the result is known, so the solver works to find the missing addend or subtrahend. In a start unknown problem, the initial addend or minuend is unknown to the solver but the second addend or

the subtrahend and the result are known so that the solver can work to find the initial number for the problem. When working to solve a problem, learners apply conceptual and procedural knowledge throughout the solving process.

The 6 word-problems used in the interview were randomly chosen as a sample of problems that learners had seen before on the formative assessments given during mathematics instruction. The word problems used in both the classroom formative assessments and the interview were reviewed for accuracy and validity by a panel of three mathematics education experts for grade-appropriate mathematics content, relevant language, and context. Table 1 below shows the interview questions answered by the learners:

Table 1. Standard interview questions

Order	Problem Type and Word problem
1	Change Unknown The soccer team had 881 raffle tickets to sell for a fundraiser. They sold some tickets during the game on Friday night. Now they have 792 tickets left. How many tickets did they sell? Result Unknown
2	Start Unknown On Sunday, 557 people attended the dinosaur exhibit at COSL. On Monday, 520 people attended the exhibit. How many people visited the exhibit over two days? Start Unknown
3	Result Unknown. The basketball team raised money for a fundraiser. They raised R359 at a car wash. Now they have R900. How much did the team have to start with? Result Unknown.
4	On Friday, 707 grade 2 learners visited COSL. A group of 390 learners had to leave on the first set of buses to return to school. How many learners left on the second set of buses?
5	Change Unknown The country fair is hoping to have 693 visitors for a Sunday morning. If 295 people need to come to the fair, how many more people need come to the fair to reach the goal of 693 visitors?
6	Start Unknown Learner Council is selling some Buckeye necklaces. They sold 661 necklaces at lunch on Friday. Now they have 763 necklaces left. How many Buckeye necklaces did they start with?

Learners had the opportunity to use paper and pencil during the interview to help them solve the problems. Base-10 blocks, base-10 charts, and blank number lines were available for learners to use, if they wanted. If a learner struggled to work through the problem, the researcher reminded the learner of these materials. As an extension to the interview, the researcher asked learners to solve problems involving four-digit and five-digit numbers as well. Learners were able to choose whether or not they wanted to attempt to solve these extension problems.

Table 2. Extension Questions

Order	Interview question
1	There are 7204 learners attending the soccer tournament. Of this group, 4638 of the learners are from Soweto. The other learners are not from Soweto. How many learners at the tournament are not from Soweto?
2	There are 23075 learners attending the soccer tournament. Of this group, 17899 of the learners are from Soweto. The other learners are not from Soweto. How many learners at the tournament are not from Soweto?
3	A school district has 9428 learners. If 5692 of the learners in the district are girls, how many of the learners in the district are boys?
4	A school district has 42103 learners. If 25738 of the learners in the district are girls, how many of the learners in the district are boys?

Each interview was video recorded and transcribed. In addition, the dialogue between the interviewer and the learners was detailed along with scanned learner work that was completed by the learner during the interview. Learners' work during the interview was further analysed using an interpretive framework-the cognition-based assessment (CBA) developed by the Battista (2012). Battista (2012) developed a five-level framework to describe the sophistication of learner understanding for various concepts within addition and subtraction.

4. RESULTS AND DISCUSSION

The results include assessments in word format for each of the 6 learners. Learners were free to choose their own operation based on their understanding of the problem. The reflections of the interviewed learners were written separately. However, some reflections that were repetitive could not be repeated for each learner. **Table 3.** summarises one of the participants, Zenzo's performance on the interview/word problems, displaying his accuracy rate and levels of conceptual understanding according to the CBA framework.

Table 3. Zenzo's Performance

ZENZO	1	2	3	4	5	6	PE1	PE2
Interview problem	1	2	3	4	5	6	PE1	PE2
Accuracy CBA	I	I	I	I	C	I	I	I
Conceptual Level	3.1	3.1	3.2	3.2	3.2	3	0	0

I - Incorrect, C - Correct

Question 1: Zenzo described the way he added 8 to 792 to get 800. That could be considered as evidence of Zenzo's understanding of number relationships. It can be suggested that Zenzo demonstrated conceptual understanding at level 3.1, which is a higher level than what was suggested by his use of the algorithm to solve the problem. However, Zenzo did not elaborate.

Question 2: Using the number line, Zenzo began with 557 on the left-hand side of the line and indicated that the numbers decrease in value as one moves horizontally from the left side to the right side on the number line. Zenzo's use of tens as benchmarks is evidence that he is operating at level 3.1 within the CBA framework. Zenzo used successive subtraction to decrease the value of the numbers on the number line, using -7 to decrease from 557 to 550 and then -30 to decrease from 550 to 520. When asked where he might find the answer to the problem, Zenzo pointed to the numbers above the number line and said that the solution is 37. Zenzo found the difference between the two numbers but the correct answer was the sum of the two numbers. Although he has used a valid strategy, Zenzo has not solved the problem given.

Question 3: In this representation, Zenzo used the typical number line format, with the lesser values represented on the left side of the number line and numbers increasing as one moves horizontally from the left side to the right side. The confusion that Zenzo showed in the number line used with the previous problem was not repeated in this problem, perhaps because there was no time element included in this problem. Zenzo wrote 359 on the left side, and then added 100 to reach 459 although he wrote +400 on the number line. He then added +51 to move from 459 to 500. This is an addition error since he could have added 41 rather than 51, indicating that Zenzo demonstrated some difficulty with the grouping of tens. Zenzo then added +400 to the 500 on the number line to get to the 900 represented on the right side of the number line. Zenzo's use of separately adding tens and hundreds suggests that he is operating at level 3.2 on the CBA framework. When asked for the solution to this problem, Zenzo replied that the answer is 851, that is, $400 + 51 + 400$. Zenzo elaborated that 4 plus 4 equals 8, underlining the two 400s represented on the number line, and then he stated that it would be 800 plus 51.

Question 4: Zenzo did not have the correct answer for the problem. However, he used elements of the number line to track his thinking about the problem. His use of combining groups of 110 suggests that he is operating at level 3.2 within the CBA framework because he is adding parts of the numbers.

Question 5: It is fascinating that Zenzo used 8 to get from 295 to 303 rather than adding 5 to get to 300. His final addition of 390 allowed him to arrive at the number 693 on the right of the number line. Zenzo gave a brief explanation of adding each amount to count up to 693. Because he is again using parts of the numbers to add, and is less focused on benchmarks of tens or hundreds, Zenzo's conceptual understanding would be categorized as level 3.2 within the CBA framework.

Question 6: In his explanation, Zenzo demonstrated conceptual understanding within level 3. His solution could be coded as 3 because he was able to correctly complete the algorithm, but it was unclear whether or not he had a clear understanding of it. However, Zenzo does not clearly use any explanation of place value, which reflected some lack of understanding.

Generally, Zenzo used the standard algorithm and the number line during the interview. He only managed to solve question 5 correctly, a change unknown problem. Zenzo did not solve any of these problem types correctly during the interview. Given his performance during the interview, it can be concluded that Zenzo has some conceptual understanding of numbers. For example, Zenzo may have demonstrated a conceptual understanding with decomposition of numbers by adding 8 to 295 to get 303 as part of his goal of reaching 693 when answering question 5. However, this knowledge appears fragile and Zenzo was not able to use it consistently to help him solve other problems accurately. However, at times, Zenzo seemed to be confused about whether to add or subtract the numbers. In total, Zenzo answered 8 questions (6 standard questions plus 2 extension questions) but could only solve 1 (12.5%) correctly. Generally, Zenzo had problems with word problems in mathematics. The subsequent sections explain the performance of the participants without going through each individual question in the interview. Mpho's performance analysis is presented in [Figure 4](#).

Table 4. Mpho's Performance

MPHO								
Interview problem	1	2	3	4	5	6	PE1	PE2
Accuracy CBA	C	C	I	C	C	C		
Conceptual Level	2	3.3	3.3	1.4	3.2	1.4		

Mpho's responses were on average coded as 3 in which it was unclear whether or not he understood the algorithm being used. Mpho's conceptual understanding was categorised as level 3 (3.2 to 3.3) for 3 of the 6 problems he solved. One solution was categorised as level 2, and 2 solutions were categorised as level 1. While Mpho was successful in finding correct answers for 5 of the 6 problems (83.3%) during the interview, it was never clearly evident that Mpho understood the conceptual foundations of the standard algorithm. Mpho used the standard algorithm to solve all of the problems during the interview with the exception of question three. In question three, Mpho attempted to use the base-10 blocks to answer the question and then switched to using a partial-sums strategy when he was unable to figure out how to regroup the zeroes from $900 - 359$. Question three is the only question that Mpho solved incorrectly, suggesting that he is very successful in finding correct answers when using the standard algorithm. The third interview question was a start unknown problem type. Despite being able to answer 83.3% of the questions, Mpho was unable to fully explain place value concepts. When describing how he solved the problems, Mpho focused on each place as an independent subtraction fact with no reference to the place value. This suggests that he does not fully understand the underlying concepts of place value involved in using the algorithm.

Table 5. Xolani's Performance

XOLANI								
Interview problem	1	2	3	4	5	6	PE1	PE2
Accuracy CBA	C	C	C	C	C	C	C	C
Conceptual Level	3.3	1.4	3.3	1.4	3.3	3	1.4	1.4

Throughout the interview, Xolani consistently used the standard algorithm appropriately to find accurate solutions. During the interview, Xolani solved 100% of the problems correctly, demonstrating an improvement. His success and ability to describe place value correctly in addition to his flexibility with describing how other strategies can be used to solve addition and subtraction problems suggests that Xolani has strong number sense and understands the standard algorithms. Research (Battista, 2001) has indicated that, when learners learn the standard algorithm for addition or subtraction early in their learning, it can interfere with their development of understanding other strategies. While Xolani appeared to understand and explain other strategies for solving problems, he demonstrated a clear preference for the standard algorithms. During the interview in the discussion about the first problem, Xolani stated that he preferred the standard algorithm because, "... it's the one that I learned first". In order to fully understand an algorithm, learners need to be at level 3.3 within the CBA framework. There was not enough evidence to state with certainty that Xolani fully understood the standard algorithm but his accuracy and ability to explain aspects of other strategies suggest that he had a strong conceptual foundation for use of algorithms.

Inga

There were 2-word format problems (total of 8 problems) for each of the three problem types. Inga solved Result Unknown, Change Unknown and Start Unknown problems correctly. During the interview, Inga exclusively used the standard algorithm to solve the problems. Inga correctly solved 2 of the 7 (28.6%) interview problems that included 1 extension question. Although she could solve the result unknown and the change unknown problem types, she had challenges on the start unknown problem type. Both of the problems that Inga got correct were solved as subtraction problems. Inga appeared to have some knowledge of the need for regrouping. The other misconception revealed during Inga's interview was the way in which she thought about whether numbers should be added or subtracted. Rather than focusing on the context of the problem itself, Inga appeared to look at the magnitude of the numbers to determine the operation that needed to be used. Inga gave an example of adding the numbers 37 and 81 because they are small numbers. Later in the interview, during her explanation of the extension problem, Inga pointed out that the numbers were too large to add and proceeded to subtract the numbers. Because Inga used subtraction to solve all of the problems in the interview, it might be concluded that Inga viewed three-digit numbers as being large.

Sibu

Table 6 summarises Sibu’s performance and conceptual levels for the interview/word problems:

Table 6. Sibu’s Performance

SIBU							
Interview problem	1	2	3	4	5	6	PE1
Accuracy CBA	C	C	C	C	C	C	I
Conceptual Level	1.4	1.4	1.4	1.4	3.3	1.4	3.3

During the interview, Sibu used the standard algorithm to correctly solve all six of the interview problems (100%). The one exception to this use of the standard algorithm was that Sibu tried to use a number line to solve the extension problem but she was not successful in her attempt. Including the extension problem, Sibu’s accuracy rate on the interview problems was 6 out of 7 (85.7%). However, by solving each of the standard interview problems accurately (100%), Sibu showed conceptual understanding of the word problems. Sibu successfully solved the result unknown, change unknown, and start unknown problem types. Sibu was very accurate in her computation, and the use of addition to check her answer assisted her to identify an error that she was able to correct in interview problem 4. Sibu’s inability to solve the extension interview problem using the standard algorithm could be a reflection of a fragile comprehension of the problem. While Sibu was able to explain the standard algorithm, it appears that her conceptual understanding of it was limited and she relied on the procedural components of the algorithm to compute an answer. However, Sibu was able to explain the procedures used in the other representations of addition and subtraction, which included the use of the number line in interview problem 5.

Table 7. Ruvimbo’s performance and conceptual levels for word problems

RUVIMBO								
Interview problem	1	2	3	4	5	6	PE1	PE2
Accuracy CBA	C	C	C	C	C	C	C	I
Conceptual Level	3.3	2.1	3.3	3.3	3.3	3.3	3.3	1.4

On the 6 standard interview problems presented in word format with three-digit numbers, Ruvimbo solved 100% of the problems accurately. Ruvimbo was very diverse in the representations she used to solve the interview problems. She often used multiple representations within the same problem as a way to check her solution. Ruvimbo applied the standard algorithm, the number line, the base-10 blocks, and a base-10 chart to solve the problems. Ruvimbo was often able to use the standard algorithm accurately but struggled when the number of digits increased to 5. Ruvimbo’s flexibility in her strategies suggests that her number sense and understanding of number relationships is strong. However, it was not always clear in her verbal explanations that she understood these relationships. For example, she described the partial differences algorithm as a “tricky” way to solve problems. Ruvimbo also said that regrouping was frustrating for her.

Summary of Results

In summary, learner accuracy for the six interview questions according to problem type is presented in [Table 8](#).

Table 8. Accuracy on interview problems by problem type

Name	Problem Type		
	Results of Unknown	Change of Unknown	Start of Unknown
Zenzo	0%	0%	50.0%
Mpho	100%	100%	50.0%
Xolani	100%	100%	100%
Inga	50.0%	0%	0%
Sibu	100%	100%	100%
Ruvimbo	100%	100%	100%

By and large, Xolani had a high accuracy rate, especially when solving the interview problems. Xolani’s reflections show that he did not understand the place value embedded within the algorithm in the questions. He could have used partial sums and partial differences alongside standard algorithms. Inga’s misconceptions about addition and subtraction situations are of immediate concern. Inga’s instruction focus could be sharpened on differentiating between addition and subtraction situations. Inga was more proficient in solving problems in word format when compared to problems in number format; so, word problems could be used as a foundation for her instruction. It would also be essential if Inga would link the

word problem with both the concrete materials used to model the problem and the mathematical symbols employed to represent the problem. Sibü used addition to check her subtraction problems. This suggests that she is not as confident when solving subtraction problems, hence, it would be prudent if Sibü's instruction focus could be sharpened on the use of partial sums and partial differences algorithms. Ruvimbo performed in the high range and solved all of the interview problems accurately. She frequently used multiple strategies when solving the interview problems. Given her success in solving word problems, helping Ruvimbo to provide context for mathematical problems could be very important in her understanding of future mathematics.

Discussions

The goal of this study was to provide insight into strategies that learners use to solve three-digit addition and subtraction problems in word format. A better understanding of these strategies can inform future classroom instruction.

Evidence of Conceptual Understanding

The Common Core State Standards for Mathematics (Common Core State Standards Initiative, 2010) expects that learners ought to use easily the standard algorithms for addition and subtraction in grade 2. Learners in this study who were able to use the standard algorithms accurately and successfully also competently evaluated and explained other methods of addition and subtraction such as the number line or the use of the partial sums or partial differences algorithm. These findings concur with those of Lee and Hwang (2022) that call for teachers to provide students with mathematical tools to support their investigation, reasoning, and justification. All learners involved in the study struggled with verbalising how they solved problems using the standard algorithm. No learner responses were categorised above level 3.3 within the CBA framework, which reflects some traces of misconceptions. Although Xolani was able to correctly solve all of the word problems presented during the interview, including the extension problems, his explanation of his work did not provide clear evidence regarding his understanding of the standard algorithm. An important part of the Common Core State Standards (2010) for Mathematics is the 'mathematical practices'. One of these practices is to reason abstractly and quantitatively. Learners involved in this study did not reference place value in their explanations although those who were successful in using the standard algorithm could identify the difference between values, such as the difference between an 8 in the tens place and an 8 in the hundreds place. Ruvimbo, for example, described the partial differences algorithm as being tricky although she was able to describe pieces of the algorithm. Her response suggests that her translation of place value within addition and subtraction algorithms is incomplete.

Misconceptions About Multi-digit Addition and Subtraction Problems

In this study, Zenzo and Inga were both low mathematics achievers. Because neither of them was very successful in solving problems, the misconceptions they demonstrated are worth considering. Zenzo struggled to use the standard algorithm accurately although he was able to solve the fifth interview problem correctly using the standard algorithm. A strength for Zenzo was his ability to decompose numbers and put them back together. Building on this foundation of number sense could be important for Zenzo. Capone et al. (2021) hence affirm that word problem solving refers to the whole process of engaging with a word problem in order to solve it. If Zenzo is provided with opportunities that help him connect his mental strategies with a representation, his thinking could be essential for his future progress in mathematics. In most cases, Zenzo's errors were related to errors in calculation rather than comprehension of the word problem because he could easily identify whether the given situation needed addition or subtraction to solve. Generally, Zenzo demonstrated partial translation of the use of a number line but was not able to use the number line to accurately solve problems. Pongsakdi et al (2016) hence assert that many students solve word problems by working instantly on the mathematical operations with given numbers without a deep understanding of the context of the word problem and the proper use of realistic reasoning.

Inga was another outstanding case due to her misconception that small numbers should be added, and large numbers should be subtracted. It is not clear how she developed this idea, but she was very confident in her explanations that the magnitude of the number determined the operation rather than the situation described in the problem. The sources of her errors seemed to be lack of reference to mediating artifacts such as number lines or other real contextual situations when dealing with addition and subtraction (Makonye & Fakude, 2016). Inga seemed obsessed with small numbers hence she ensured that the answers should remain small by subtracting large numbers and add small numbers. Inga's translation of

addition and subtraction situations was not a partial transference but rather a completely incorrect translation that impacts her understanding of addition and subtraction.

5. CONCLUSION

In this study, some inferences can be made about a learner's conceptual understanding based on their use of other representations such as number lines and base-10 blocks. However, there was no explicit evidence within this study to support categories learner above level 3.3. Applying literacy skills might help learners to visual the situations they are attempting to solve with mathematics. Regarding literacy skills, Lee and Hwang (2022, p.271) emphasized flexibility and variability in meaningful use of representations among contextual, visual, verbal, physical, and schematic (or symbolic) representations. The visual representation retains most of the detailed information of the original contexts and clearly. While the sample size is not large enough to draw generalization for mathematics instruction, the qualitative data provide a snapshot of the strategies learners use when engaging in the problem-solving process. Learners demonstrated a strong preference for paper and pencil methods when solving problems in word format. These results are similar to the conclusions of Koedinger and Nathan (2004) in which they suggest that high school learners are more successful in solving word format problems when compared to symbol format problems. The learners in the current study were younger than the learners involved in the work of Koedinger and Nathan, but the results suggest a similar pattern. The pattern allowed us to hypothesize that early identification of the difficulties and intervention strategies suggested could help to overcome future impediments in understanding word problems in mathematics (Capone, Filiberti & Lemmo, 2021). Allowing time within classroom instruction for classroom teachers to interview learners and uncover potential misconceptions remains an important component in teaching mathematics. Ensuring that learners have a firm foundation of mathematics during primary school may enhance their future success in mathematics. Since most learners faced challenges developing place value concepts, the use of base-10 blocks to model the partial sums and partial differences algorithms would help most of the learners to develop conceptual understanding of addition and subtraction of word problems in mathematics. Lee and Hwang (2022) hence aver that teachers should provide learners with mathematical tools to support their investigation and reasoning in mathematics.

CONFLICT OF INTEREST

There are no conflicts of interest declared by the authors.

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