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RESEARCH ARTICLE

Algebraic Thinking Profiles of Junior High Schools' Pupil in Mathematics Problem Solving

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ABSTRACT

Algebra is one of concept that must be learned by pupils. It is because the algebraic concept can be used in all areas of mathematics. One of the ways that is used to develop pupils' algebraic abilities is to think algebraically. While one of the ways to develop pupils' algebraic thinking skills is to adapt pupils with mathematical problem solving. The purpose of this study is describing algebraic thinking profiles of junior high school pupils in mathematical problem solving. The description of pupils' algebraic thinking profiles is explained based on six indicators namely generalization, abstraction, analytic thinking, dynamic thinking, modeling and organization. This research is a qualitative study using test and interview methods. The research subjects consisted of one student in each student with high mathematical abilities, medium mathematical abilities and low mathematical abilities. The results showed that there were differences in algebraic thinking of junior high school pupils in solving mathematics in terms of mathematical abilities. Based on research data shows that pupils with high mathematical abilities always think algebraically in each problems solving are given, while students with medium and low ability do not always think algebraically in each of problem solving.

1. INTRODUCTION.

Algebra is one of the mathematics subject that is learned in school and also include in one of the concepts students must learned when studying. Algebra is also known as a branch of mathematics that studies structure, relationships and quantity. According to Kusaeri (2012), algebra is one branch of mathematics that is important in shaping pupils' mathematical characters. The use of symbolic in solve problems is a characteristic of algebra, so algebra is very important to be learned. This is as revealed by NCTM (National Council of Teacher of Mathematics) that "algebraic competence is important in adult life, both on the job and as preparation for post secondary education".

A study is conducted by Windsor (2010) revealed that algebraic thinking appear after there was a gap between arithmetic and algebra, as is known that learning algebra makes students in an unknown quantity, but as if the quantity is known. This is also revealed by Kieran (2004: 140) that in the transition from arithmetic to algebra, pupils will need some adjustments, including pupils who are quite capable of arithmetic. Another prespective explains that algebra is difficult because its viewed and taught as meaningless manipulation of abstract systems of symbols (Arcavi, 2008; Saul, 2008). Again, discussed by Welder (2012), asserts that difficulties with algebra persist because of misconceptions that block algebraic understanding. Based on Welder's theory, previous understanding or misuderstandings "can stand in the way of acquiring new knowledge when the new learning content is incompatible with what is already known" (Christou, Vosniadou, & Vamvakoussi, 2007, p.285).

So that, for teachers this is a serious challenge that must be solved and understood, so there are differences in thinking between arithmetic and algebra, so that in learning algebra pupils must be equipped with different ways of thinking from arithmetic thinking, with the aim of understanding algebra as well as possible , remembering algebra and algebraic thinking can be found throughout the area of mathematics and is important enough to make mathematics useful in their life.

Luis Radford (2006) formulates the characteristics of algebaric thinking as follows :

1. One deals with a sense of indeterminary that is proper to basic algebraic objects such as unknowns, variables and parameters
2. Indeterminate objects are bandled analitycally
   The peculiar symbolic mode that it has designate its objects

With algebra pupils are trained to think critically, creatively, reason, and think abstractly. One of the way to develop pupils algebraic abilities is thinking algebraic. According to Lew (2004) algebraic thinking is a mental activity consisting of several thinking activities, there are:

1. Generalization
   Generalization is a process to find a pattern or a form
2. Abstraction

Abstraction is a process to extract mathematical objects and relations based on the generalization

3. Analytic Thinking
   The process of finding some unknown value requested in the expression written in terms of the unknown value
4. Dynamic thinking
   Thinking which is related to the dynamical manipulation of mathematical objects. Dynamic thinking could be developed by hypothetical deduction and the trial and error strategy for monitoring and controlling the dependent action for each of changing variables.
5. Modeling
   Modelling is a process to represent the complex situation using mathematical expressions, to investigate the situation with a model, and to draw some conclusions from the activities.
6. Organizations
   Organization promotes combinatorial thinking for finding all of the independent variables, which is very important to many problem-solving activities. By sorting ang organizing data by making a table, a whole picture about the problem situation and the relation between conditions of the problem can be grasped and the relation between an independent variable and the corresponding dependent variable can be controlled more easily.

Walle et al. (2013: 259) explained that there are five forms of algebraic thinking, including: 1) Generalization from arithmetic, 2) Use of meaningful symbols, 3) Study of number system structure, 4) Study of patterns and functions, 5) Process of mathematical modeling

Indicators of algebraic thinking are generally formulated by researchers based on the opinion of Lew (2004) about the type of algebraic thinking without the last type of algebraic thinking, organization. Pupils are said to think algebraically if the process of problem solving shows at least one of the indicators of algebraic thinking: 1) generalizing, that expressing patterns or formulating general symbolic, 2) doing abstractions, that extracting objects and mathematical relationships based on generalizations, 3) analytical thinking, that solving equations to determine unknown quantity, 4) dynamic thinking, that doing dynamic manipulation of mathematical objects, and 5) modeling, which represents problems in mathematical models

Discussing about algebraic thinking, of course can not be separated from pupils' ability to solve mathematical problems. This accordance with the opinions of Norton and Windsor (2011) that "by developing algebraic thinking using a problem solving approach, student develop a way of thinking builds from their own mathematical understanding and provides an entry point into more sophisticated mathematics". That by developing algebraic thinking using a problem solving approach, pupils can develop ways of thinking that are built from their own mathematical understanding and provide a more sophisticated pathway to mathematics. Kristen Herbert and Rebecca Brown (2000) explain the algebraic framework in solving problems as follows:

"algebraic thinking is using mathematical symbols and tools to analyze different situatons by:

Extracting information from a situation

Representing that information mathematically in word, diagram, tables, graphs, and equations, and

Interpreting and applying mathematical findings such as solving for unknown, testing conjectures, and identifying functional relationships to the same aituations and the new related situation".

That algebraic thinking is thinking by analyzing a situation with tools and symbols of mathematics through three activities. The three activities are: 1) extracting information from the situation by analyzing the situation by determining useful and useless information that can be used to deal with the situation is given, 2) restating the information mathematically in the form of words, diagrams, graphs, tables , and equations, and 3) interpretting and applying mathematical findings, such as finding solutions to the unknown, testing suspicions, and identifying functional relationships for the same situation and related new situations.

In this study researchers adopted an algebraic framework of mind in problems solving and combined with previous opinions. This study formulated indicators of algebraic thinking that might arise in problems solving, as follows :

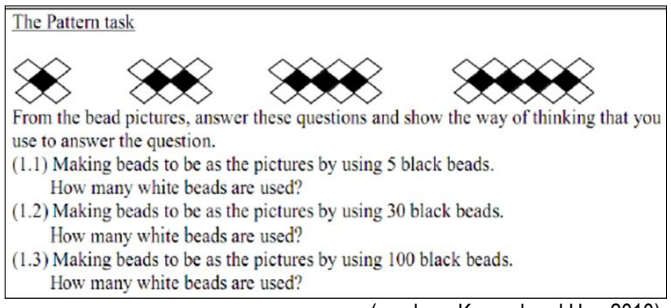
Table 1. Indicators of algebraic thinking in problem solving

Problem solving activities	Indicators of algebraic thinking	Example of activity
Extracting information from a situation	Understanding the use of symbols in the form of letters, images, or word to represent variables as generalizations of numbers or variables as something unknown (unknown quantity	Understand what is asked on a question that involves the use of symbols in the form of letters
	Determine the previous term or the next term by analyzing the relationship between numbers	Use the information is given on the question and deep exploration of the information to see the relationship between numbers
Representing information mathematically	Represents the relationship between numbers in pictures, words or algebraic forms to express a number pattern	Stated pattern with words
		Make images for representation
	Arrange the equations to express the relationship between quantities	Write equations to describe the problem to be solved
Interpretting and applying mathematical findings	Represents the relationship between numbers in pictures, words or algebraic forms express a number pattern	Write algebraic forms based on patterns that have been found
	Determine the previous term or the next term by analyzing the relationship between numbers	Determine the previous term or the next term based on the relationship between numbers
	Determine the value of a variable as something unknown	Complete the equestion

Pupilts' abilities in solving mathematical problems also vary based on the level of mathematical abilities. The level of mathematical abilities can be divided into three, namely pupils with: 1) high ability, 2) medium ability, and 3) low ability. In general students who have high abilities in mathematics are more able to solve problems, especially problems related to algebraic material. this is because with solving a problem pupils with high ability have an understanding of algebraic concepts as well as knowledge about strategies in solving better than the other groups. This understanding and ability will be the main basis in developing pupils' algebraic thinking processes in problem solving.

The problems in this study are presented in the form of questions developed by adapting the problems used in Natcha Kamool and Yeah Ban Har's research (2010:290), as shown in the

following figure:



(sumber : Kamool and Har, 2010)

Figure 1. Example of a problem related to a number pattern

2. RESEARCH METHOD

This approach used in this study is a qualitative approach. The purpose of this study is to describe the profile of algebraic thinking in junior high school pupils in mathematical problems solving based on mathematical abilities. This research was conducted in class VII of SMP Negeri 1 Jombang. The subjects in this study consisted of three students, each of whom had high mathematical abilities, medium mathematical abilities and low mathematical abilities.

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Table 1. Criteria for Grouping pupils Mathematical Abilities

Score	Level of Ability
s ≥ 80	HIGH
80 > s ≥ 70	MEDIUM
s < 70	LOW

Instruments in this study consisted of the main instruments and supporting instruments. Supporting instruments are namely: 1) Mathematical Abilities Test Questions (TKM) which consisting of five item description questions which arranged by adapting the mathematics problems of Ujian Nasional SMP for about five questions, 2) Problem Solving Test questions (TPM) ,which consist of description questions to explore the emergence of pupil algebraic thinking profiles, and 3) interview guidelines which used are task-based interviews, which aim to assist researchers in revealing in depth how the pupils' algebraic thinking profiles in solving problem solving test questions are given.

Data collection in this study was conducted by combining two main activities, namely tests and interviews. The types of tests are: 1) mathematical ability tests used to classify pupils as the basis for selecting research subjects, and 2) problem-solving tests used to obtain data about pupils algebraic thinking profiles in problem solving, and 3) interviews used to obtain data clearer rather than changing pupils answers to be correct about pupils algebraic thinking.

Technical data analysis in this study was carried out through:

1. Data reduduction
   Data reduction is a form of analysis which used to discard of unnecessary data and organize raw data obtained from the field
2. Presentation of data
   Data from interviews that have been transcribed are presented in conversation codes so that readers can understand the results of the interview easily
3. Make Conclusion
   Make conclusions is based on the data that has been analyzed, including data on problem solving test results and interview results. Making conclusions are presented in the form of concise and concise descriptions but contain broad understanding

3. RESULTS AND DISCUSSION

Collection of data in this study was in SMP Negeri 1 Jombang. TKM is given to class VII pupils. The result of the TKM are analyzed and grouped into certain mathematical abilities. The TKM results data are presented in the following table

3.1 Tables

Table 2. TKM Results and Grouping of Pupils based on Mathematical Abilities

No.	Name's Code	Value	Pupils mathematical abilities
1	A-01	70	MEDIUM
2	A-02	68	LOW
3	A-03	75	MEDIUM
4	A-04	71	MEDIUM
5	A-05	75	MEDIUM
6	A-06	55	LOW
7	A-07	72	MEDIUM
8	A-08	75	MEDIUM
9	A-09	65	LOW
10	A-10	71	MEDIUM
11	A-11	85	HIGH
12	A-12	75	MEDIUM
13	A-13	74	MEDIUM
14	A-14	70	MEDIUM
15	A-15	80	HIGH
16	A-16	72	MEDIUM
17	A-17	55	LOW
18	A-18	83	HIGH
19	A-19	70	MEDIUM
20	A-20	70	MEDIUM
21	A-21	82	HIGH
22	A-22	80	HIGH
23	A-23	75	MEDIUM
24	A-24	81	HIGH
25	A-25	73	MEDIUM
26	A-26	80	HIGH
27	A-27	70	MEDIUM
28	A-28	70	MEDIUM
29	A-29	73	MEDIUM
30	A-30	72	MEDIUM
31	A-31	65	LOW
32	A-32	70	MEDIUM

Based on TKM results data by considering the results of TPM and good communication, three pupils were chosen as the research pupils. The three research pupils can be seen in the following table

Name's code	Score of TKM	Mathematical Abilities
A-11	85	HIGH
A-03	75	MEDIUM
A-17	55	LOW

Furthemore, the TPM results data and interviews were analyzed using steps to solve problems according to Herbert and Brown (2000), so that the following results were obtained:

Algebraic Thinking profiles of pupils high mathematical abilities

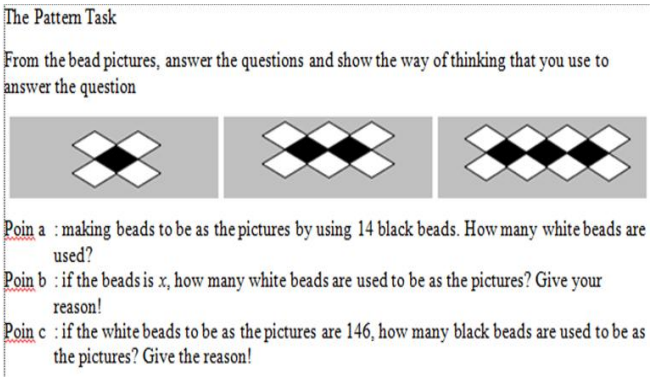
In solving mathematical problems, KT pupilss do each stage of problem solving. And in each stage of problem solving, the algebraic indicator always appears. Algebraic indicators that arise are generalization, abstraction, dynamic thinking, modeling and analytic thinking.

Algebraic Thinking profiles of pupils medium mathematical abilities

In interpreting and applying mathematical findings, indicators of analytic thinking in pupils with medium mathematical abilities only appear in one TPM. This is because pupils with mathematical abilities are not using mathematical findings in solving problems, but use comparative principles of value. Thus, indicators of algebraic thinking that arise at the stage of extracting information from the situation and restating the information mathematically are generalization, abstraction, dynamic thinking and modeling

Algebraic Thinking profiles of pupils low mathematical abilities

In presenting information mathematically, pupils with low mathematical abilities feel difficult to find common patterns and express those patterns in the form of algebra. However, pupils with low mathematical abilities can represent patterns with words and understand symbols as unknown. In interpreting and applying mathematical findings, pupils with low mathematical abilities do not apply the mathematical findings to the two TPM, so that indicators of algebraic thinking do not arrise. Thus indicators of algebraic thinking that arise at the stage of extracting information from the situation and restating the information mathematically is dynamic thinking and abstraction


The Pattern Task

From the bead pictures, answer the questions and show the way of thinking that you use to answer the question

Poin a : making beads to be as the pictures by using 14 black beads. How many white beads are used?

Poin b : if the beads is x, how many white beads are used to be as the pictures? Give your reason!

Poin c : if the white beads to be as the pictures are 146, how many black beads are used to be as the pictures? Give the reason!

Figure 2. Problem solving test

4. CONCLUSION

Based on the analysis of research data and the discussion that described above, a conclusion can be drawn as follows:

1. Algebraic thinking profiles of pupils high mathematical abilities
   In solving problems, pupils with a high mathematical abilities always think algebra at each stage of problem solving for all problems given. Based on the problem solving that is done on each point of the problem given can be obtained a general description of the profile of pupils high mathematical abilities. Pupils with this ability think algebra at the stage of extracting information by analyzing the relationship between numbers by exploring information in the image that displayed on the question. Furthermore, at the stage of restating, pupils think algebra by stating the results of the analysis in the form of overall representation in the form of patterns or using images or using words or algebraic forms. Finally, at the stage of applying and interpreting the mathematical findings, pupils with high mathematical abilities apply rules or patterns to provide solutions to problems.

   Based on indicators that are met ,pupils with this ability involve five thinking activities in algebraic thinking namely generalization (finding general patterns or shapes by representing relationships from patterns into forms or algebraic expressions), abstraction (extracting mathematical objects and relationships based on generalization), modeling (represent problems in mathematical models), dynamic thinking (use of variables in performing dynamic manipulations of mathematical objects), and analytic thinking (solving equations to determine the value of a variable as something unknown).

2. Algebraic thinking profiles of pupils medium mathematical abilities
   In solving the problem, pupils medium mathematical abilities do not always think algebra at each stage of solving the problem. Only on points c pupils think algebra for each stage of problem solving. At this stage pupils medium mathematical abilities think algebraically in extracting information from a given problem by analyzing relationships between numbers. Furthermore pupils medium mathematical abilities think algebraically in restating mathematical information through words that express patterns or rules in general. Furthermore, pupils medium mathematical abilities think algebraically in applying and interpreting mathematical findings by applying the rules obtained in the previous stages in solving the problem.

   Based on fulfilled indicators, students with this level of ability fulfill four activities of thinking in algebraic thinking, namely generalization (finding general patterns or shapes by representing relationships from patterns into forms or algebraic expressions), abstraction (extracting mathematical objects and relationships based on generalization), modeling (representing problems in mathematical models) and dynamic thinking (use of variables in performing dynamic manipulations of mathematical objects).

3. Algebraic thinking profiles of pupils low mathematical abilitiesIn solving problems, pupils low mathematical abilities do not always think algebraically. At point c pupils do not think algebra for each stage. Whereas in points a and b pupils with this ability do not think algebraically at the stage of applying and interpreting mathematical findings.

Based on this, indicators that are filled with pupils low mathematical abilities involve two algebraic thinking activities, namely dynamic thinking (analyzing the relationship between numbers in a pattern to determine the next term), and abstraction (understanding the use of symbols in letters, images or words to represent variable as something unknown).

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