

Algebraic Reasoning of Students with Logical-mathematical Intelligence and Visual-spatial Intelligence in Solving Algebraic Problems

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ABSTRACT

This study aims to describe the algebraic reasoning of students with logical-mathematical Intelligence and visual-spatial Intelligence in solving algebraic problems. This research is a qualitative descriptive study. The subjects of this study were students of eighth grade students of junior high school totaling 35 students. The results of the research data were analyzed by describing the algebraic reasoning of students with logical-mathematical Intelligence and students with visual-spatial Intelligence on each of the defined indicators. Data collection techniques are carried out by tests, observation, interviews, and documentation. The results showed that the student with logical-mathematical Intelligence and visual-spatial Intelligence in pattern seeking indicators, were able to identify, represent what was known and asked in the problem and find the constituent elements of the pattern. In the pattern recognition indicator students with logical-mathematical Intelligence find a relationship between elements and the similarity of the relationship of each element by thinking about it and accompanied by logical reasons. Students with visual-spatial Intelligence find a relationship between elements and the similarity of relations between each element by conducting experiments by writing down each process completely. In the generalization indicator, students with logical-mathematical Intelligence and students with visual-spatial Intelligence are able to model the situation or problem given and solve it correctly, and they are able to find a general rule that can be used to solve problems.

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1. INTRODUCTION.

Mathematics is one of the most challenging of all subjects and it is often a difficult subject for students (Hunter, 2006). One of the causes of these difficulties is the understanding of symbols, variables, which are in algebraic material. Though mastery of algebraic material is an important competency for middle school students.

Algebra is important in everyday life, along with the increasing demand for workers in the fields of science, technology, engineering, and mathematics that require algebra (Lacey, 2017). Algebra is used to generalize about quantities, relations, functions, model situations and solve problems (Knuth et al., 2016; Watson 2009). Algebra is usually associated with solving systems of equations, finding the value of something unknown, and solving a problem using symbols (Andriani, 2015). A good understanding of quantitative relationships and relations is related to success in using algebra (Watson, 2009). In Indonesia, algebraic material was introduced in mathematics learning at the elementary level.

Algebraic material is difficult for class VIII students in Indonesia, Ontario, Midwestern and Malaysia. The research conducted by Egodawatte (2011) in Ontario and Bush (2011) in the Midwestern towards class VIII junior high school students stated that there were many student errors and misconceptions in solving algebraic problems. Similar research on the difficulties and misconceptions of

39 grade VIII students in Sabah, East Malaysia, showed that student difficulties were in five parts: (1) understanding of variables, (2) manipulation of variables, (3) using manipulation rules to solve equations, (4) using algebra knowledge and ways to form equations, and (5) generalize a pattern (Chow, 2011).

In addition, research on 39 students of eight grade in Malang confirmed that students did not understand the meaning of variables, made procedural errors, and made mistakes in solving similarities in algebraic material. The reason is failure to associate questions with concepts that have been studied before, inability to imagine about the possibilities of variable values, and lack of mastery of material (Ramadhani, 2015). Based on some of these studies, it can be concluded that junior high school students still have difficulty in solving algebraic problems.

The difficulties and misconceptions found are mostly related to understanding the meaning of variables, using rules of manipulation, and formulating and making generalizations. This is related to students' reasoning processes related to algebra or commonly called algebraic reasoning. Algebraic reasoning for each student in solving algebraic problems, of course, has different levels according to the intelligence of each student. Normal people have at least eight types of intelligence namely linguistic intelligence, mathematical logical-mathematical Intelligence, musical intelligence,

kinesthetic intelligence, spatial visual-spatial Intelligence , intrapersonal and interpersonal intelligence, existential intelligence and natural intelligence (Gardner, 2011, Armstrong, 2003).

The level of individual strength in various intelligences with how a concept is taught will determine how well the individual understands a concept. Some intelligence such as mathematical logical-mathematical Intelligence, existential intelligence, intrapersonal intelligence, and spatial/visual-spatial Intelligence are very positively correlated with solving mathematical problems (Rahbarnia, 2014). Besides that problem solving ability is influenced by linguistic, logical-mathematical, and visual spatial intelligence (Asyrofi & Junaedi, 2016). In the process of problem solving, students must be able to make connections between known concepts, experiences and new concepts, and determine a suitable strategy that can be applied. It really depends on the students' reasoning abilities themselves. Based on these studies which have positive correlations and appear in the previous two studies are mathematical logical-mathematical Intelligence and visual spatial. Therefore, in this study specifically will discuss the algebraic reasoning of students with Logical-Mathematical intelligence and Spatial-visual-spatial Intelligence.

2. RESEARCH METHOD

The research is qualitative descriptive. The described data is the algebraic reasoning of students with logical-mathematical Intelligence and visual-spatial Intelligence in solving algebraic problems. The subjects of this study were eight grade student totaling 35 students. All students are given a multiple intelligence questionnaire, so that students are divided into eight groups, namely students with logical-mathematical Intelligence , interpersonal intelligence, intrapersonal intelligence, musical intelligence, kinesthetic intelligence, naturalist intelligence, visual-spatial Intelligence , existential intelligence. Furthermore, the subject of this study is students with logical-mathematical Intelligence and visual-spatial Intelligence . This is based on the research objectives.

Student algebraic reasoning data was collected using test techniques and interviews. Tests are given to all students. Whereas interviews were conducted on representatives of each category needed in this study, namely logic intelligence and visual-spatial Intelligence . The instruments used in this study were algebraic reasoning tests and interview guidelines. This test is in the form of a description question which aims to find out the algebraic reasoning abilities of students in solving algebraic problems. This test instrument is prepared based on predetermined algebraic reasoning indicators (Herbert and Brown, 2000), namely pattern seeking, pattern recognition, generalization. The next instrument is the interview guide. Interview guidelines are used to dig deep information about algebraic reasoning of students with multiple intelligence in solving algebraic problems.

The data analysis technique used in this study is a descriptive qualitative analysis technique. The analysis used is by describing the algebraic reasoning of students with logical-mathematical Intelligence and visual-spatial Intelligence to be seen and compared with the achievement of indicators of algebraic reasoning which are accompanied by logical reasons. Grouping students into logical-mathematical Intelligence and visual-spatial Intelligence is determined based on the questionnaire that has been given before the test of algebraic reasoning to students. While data obtained based on the results of interviews were analyzed by

reducing data, presenting data, and drawing conclusions. This information is used as additional supporting data for algebraic reasoning of students with logical-mathematical Intelligence and visual-spatial Intelligence that have not been described through written tests.

3. RESULTS AND DISCUSSION

The participants in this study were 35 students in eight grade, consisting of 14 male and 21 female. Age between 13-14 years. The selection of eight grade junior high school students was based on the reason that eight grade students had received basic algebraic material in seven grade. Purposive selection is intended to select subjects based on specific characteristics according to the research objectives. These special features are in the form of active students in the class and heterogeneous student abilities.

In this study, researchers gave a multiple intelligence questionnaire adapted from McClellan, et al. (2008). Based on the results of the questionnaire, students were divided into eight groups namely 7 students with logic intelligence, 4 students interpersonal intelligence, 4 students intrapersonal intelligence, 10 students of 10 musical intelligence, 4 kinesthetic intelligence students, 1 naturalist intelligence, 3 visual-spatial Intelligence students, and 2 intelligence students existential. Furthermore, the subjects of this study were 7 students with logic intelligence and 3 students with visual-spatial Intelligence . This is based on the research objectives. Afterwards to find out the algebraic reasoning of students, the researcher gave an algebraic reasoning test that contained contextual algebraic questions. Algebraic reasoning data in this study were analyzed based on algebraic reasoning indicators adapted from three stages of algebraic reasoning in solving problems proposed by Herbert & Brown (2000).

The following is an algebraic reasoning test given to students.

1. Ms. Ani is a housewife who often buys fruits for her family. Mrs. Ani bought 2 apples and 14 oranges for the first bag. Then the second bag contains 5 apples and 28 oranges. In the 3rd bag there are 10 apples and 42 oranges.
 - a. Purchasing these fruits forms a pattern so we can find out how many apples and oranges Bu Ani bought. How many apples and oranges did you buy in the 4th bag?
 - b. How many apples and oranges did Mrs. Ani buy in the 7th bag? Explain how you found it!
 - c. Determine an equation that can help you find the number of apples and oranges in each bag! Explain how you found the equation.
2. Mrs. Reni is a businessman who sells fruits. Mrs. Reni usually wraps the fruits with a certain pattern. If Mrs. Reni determines a certain price for apples and oranges, then in the first bag containing 2 apples and 14 oranges sold for Rp.57,000.00 and in the second bag containing 5 apples and 28 oranges sold for Rp.121,500 , 00.
 - a. What is the price of apples and oranges sold by Mrs. Reni in the 3rd bag?
 - b. What is the price of apples and oranges sold by Mrs. Reni in the 6th bag?
 - c. What is the price of apples and oranges sold by Mrs. Reni in the kth bag?

3.1 Algebraic reasoning students with logical-mathematical Intelligence in solving algebraic problems.

1. Patern seeking

In completing the algebraic reasoning test, students with logical-mathematical Intelligence (SL) easily understand what is asked in the problem. This can be seen from the results of his work, he wrote down any information that is known and asked in the question. This shows that SL is able to identify things that are known and asked in the questions given. In addition, in the process of solving the problem, SL writes the numbers in sequence according to the number of apples and oranges in the order of the bags, then connects them using a chart. This shows that SL represents what is known in the form of symbols and charts.

2. Pattern recognition

In the process of solving questions 1a and 1b, SL wrote down the solution to the problem with rules or equations that could help him find the number of apples and oranges in the 4th bag and the 7th bag (Figure 1). It turns out that in the number 1a settlement process, SL has immediately thought of getting a rule so that it makes it easier to get the number of apples and oranges in any order. This shows that SL is able to find the constituent elements of a pattern, while being able to make a connection between two quantities.

Handwritten work for SL showing calculations for apples and oranges in bags. The work is divided into two columns. The left column shows calculations for apples (KA) and oranges (JA) for bag 4 and bag 7. The right column shows calculations for apples (KA) and oranges (JA) for a general bag k.

$$\begin{array}{l} = n \cdot n + 1 \\ = 4 \cdot 4 + 1 \\ KA = 17 \text{ apel} \\ = 14 \cdot n \\ = 14 \cdot 4 \\ KA = 56 \text{ Jeruk} \end{array} \quad \begin{array}{l} k \cdot 7 = \text{apel} = n \cdot n + 1 \\ = 7 \cdot 7 + 1 \\ = 50 \text{ apel} \\ \text{Jeruk} = 14 \cdot n \\ = 14 \cdot 7 \\ = 98 \text{ Jeruk} \end{array}$$

Figure 1. SL Settlement in finding the number of apples and oranges

The following is an excerpt from the interview with SL in finding general rules:

- R: What about solving problem 1b?
- SL: When solving on 1a, I also searched for the formula too, so for 1b formula is the same.
- R: it means that, after working on question 1a, you immediately sought a solution for 1c? How do you find the general rules?
- SL: Yes. I solved on 1c first. At first I tried using numbers 2 and 3. For example, number 2, if multiplied by the number itself becomes 4, it means less than 1 (with the number of apples in the second bag), then added 1 to 5 equals the number of apples in the bag 2. Then also for number 3. I multiply number 3 with the number itself then add 1, then get 10 equal to the number of apples in the 3rd bag in the problem. So I found the equation to find the number of apples is $n \cdot n + 1$. Then for the number of oranges always added 14 or multiplied directly between the order of bags with 14. So I found the equation to find the number of oranges is $14n$.

In the interview, it was seen that SL conducted experiments to find relationships between patterning elements, found relationships and similarities in relation to each element. The pattern is to determine the number of apples with 14. Then when the researcher asked what could prove that the conjectures made by SL were

correct. SL tests the general rules with numbers 1, 2 and 3, then matches the number of apples and oranges known in the question. SL believes that the similarities he has made prove the truth. This shows that SL proves the truth of the conjecture.

3. Generalization

SL has proven that the conjecture that he has made is correct, so it can be said directly that SL has determined the general equation in determining the number of apples and oranges. SL states that to find the number of apples and oranges in the first bag is $14 \times n$ for oranges, and $n(n + 1)$ for apples.

Then when finishing number 2, that is to determine the total price of apples and oranges in the 3rd bag, 6th bag, and the kth bag. SL completes it using the elimination method on SPLDV. In this settlement process, SL did not write it on the answer sheet but directly by looking at the problem, SL told how he got the price of 1 apple and 1 orange using his logic. After getting the price of 1 apple and 1 orange in a row, namely 7,500 and 3,000. SL determines the total price in the 3rd bag is $(7,500 \times 10) + (3,000 \times 42) = 201,000$. Furthermore, for the total price in the 6th bag is $(7,500 \times 37) + (3,000 \times 84) = 529,500$.

To find the general rule of the total price equation in the k-bag. SL immediately writes it in the form $((n \cdot n + 1) \times 7,500) + (14 \cdot n \times 3,000)$. This shows that SL determines general rules or equations used in solving problems.

3.2 Algebraic reasoning students with visual-spatial Intelligence in solving algebraic problems.

1. Patern seeking

In completing the algebraic reasoning test, students with Visual-spatial Intelligence (SV) easily understand what is ordered in the problem. This can be seen from the results of her work by completing a list of what is known and asked in the question. This shows that SV is able to identify things that are known and asked in the questions given. In addition, in the process of resolving the problem, SV wrote down the process of solving the problem sequentially in the form of a table according to the number of apples and oranges in each bag. This shows that SV represents what is known in the form of symbols and tables.

2. Pattern recognition

In the process of solving questions 1a and 1b, SV writes the solution to the problem using the equation that she got. (Figure 2). It turns out that in the number 1a settlement process, SV has immediately sought a rule to make it easier to get the number of apples and oranges in any order. This shows that SV is able to find the constituent elements of a pattern, while being able to make a connection between two quantities.

Handwritten work for SV showing calculations for apples and oranges in bags. The work is divided into two columns. The left column shows calculations for apples (KA) and oranges (JA) for bag 4. The right column shows calculations for apples (KA) and oranges (JA) for a general bag k.

$$\begin{array}{l} a) \text{ Ditanya: Berapa banyak apel \& \text{jeruk di kantong ke 4?} \\ \text{Pijawab:} \\ \text{apel} \\ U_n = n^2 + 1 \\ U_4 = 4^2 + 1 \\ = 16 + 1 \\ \text{jeruk} \\ U_n = 14n \\ U_4 = 14 \cdot 4 \\ = 56 \end{array}$$

Figure 2. SV Settlement in finding the number of apples and oranges

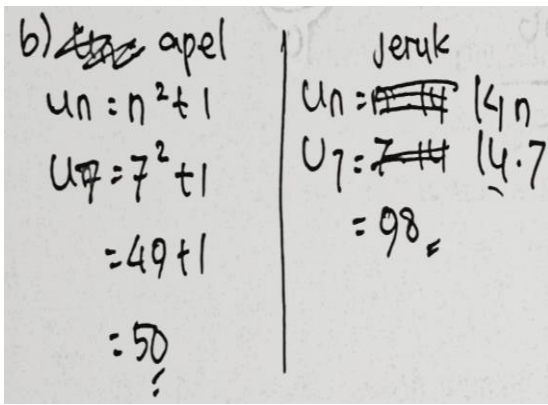


Figure 3. SV Settlement in finding the number of apples and oranges

In solving question number 1, SV immediately looked for answers to questions number 1c. Following is results of SV for 1c..

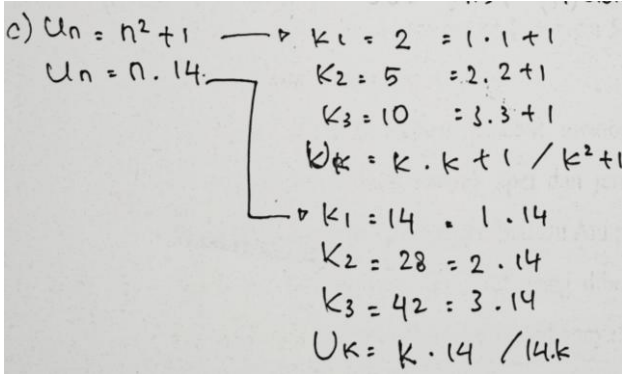


Figure 4. SV Settlement in finding general rules.

The results of test and interview conducted, it appears that SV made several attempts to test the rules she got in the first, second and third pockets. All of these experiments are used to find relationships between the constituent elements of a pattern, find relationships and relationship similarity to each pattern constituent element. So SV finds the number of apples is the result of the square of the order of pockets added to 1 and for the number of oranges is the multiplication between the order of bags with 14.

Next when the researcher asked what can prove that the rules made by SV are correct. SV replied by testing the general rules with numbers 1, 2, and 3, according to the order of the pockets known and she found according to the number of apples and oranges contained in the question. SV believes that the similarities she has made are proven true. This shows that SV was able to prove the truth of the conjecture.

3. Generalization

SL has proven that the rules he has made are correct, so it can be directly said that SV has determined the general equation or rule in determining the number of apples and oranges. SV states that to find the number of apples and oranges in the k-bag is $n(n + 1)$ and $14 \times n$.

Then when finishing number 2, that is to determine the total price of apples and oranges in the 3rd bag, 6th bag, and the kth bag. SV resolves it using the elimination method and substitution. In this settlement process, SL is complete on the answer sheet (Figure 4).

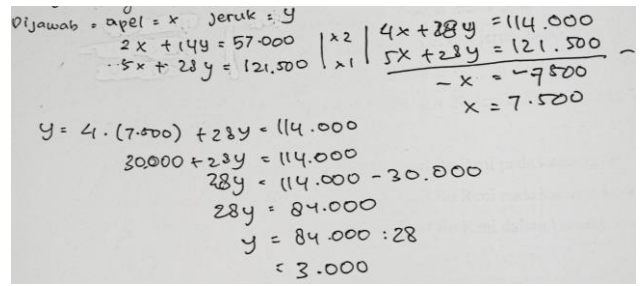


Figure 5. SV solution in modeling the problem and solving it.

In this settlement process, it can be seen that SV models the situation or problem given in the form of a mathematical model. SV makes a mathematical model according to the number of apples and oranges and their prices in the first and second pockets.

To find the general rule of the total price equation in the k-bag. SV directly writes it in the form $((n \cdot n + 1) \times 7,500) + (14 \cdot n \times 3,000)$. This shows that SV determines general rules or equations used in solving problems in algebraic form. The general rule is obtained from summing the prices of the two constituent elements in the kth bag because each bag contains both of these elements.

4. CONCLUSION

Based on the results of the research and discussion that has been explained, it can be concluded that the indicators of algebraic reasoning, the first pattern seeking students with logical-mathematical Intelligence and visual-spatial Intelligence are able to identify things that are known and asked about problems, represent them in word form, tables, symbols and charts, and are able to find the constituent elements of the pattern. In the indicator of pattern recognition of students with logical-mathematical Intelligence and visual-spatial Intelligence, they carry out experiments to find relationships between pattern compilers, and find similarities in relationships in each pattern building element. Furthermore, the last indicator of generalization of students with logical-mathematical Intelligence is able to solve mathematical models with logical processes so that they do not write down the solutions in the answer sheet, and students with visual-spatial Intelligence complete mathematical models by writing down the complete process on the answer sheet. To determine the general rules used in solving problems, students with logical-mathematical Intelligence and visual-spatial Intelligence are able to solve them correctly and are able to prove that the general rules obtained are correct.

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