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Research Article

Mathematics communication as an alternative to overcome the obstacles of undergraduate students in mathematical proof

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ABSTRACT

Experiences at the educational personnel education institution, Department of Mathematics Education Institute of Teacher Training and Education Pontianak (IKIP) shows that students often had difficulty in proving propositions (theorems in mathe-matics). The alternative offered through this study was to develop students' abilities in proof through mathematical communication. The study method used in this study was qualitive research, case study. The subjects used in this study consisted of two un-dergraduate students who had been taught real analysis introductions and. The data collection tool used was tests and interview. Data analysis techniques used was de-scriptions: data reduction, data display and conclusion. Based on the results of re-search, study of theory and discussion, it could be concluded in this study that the student obstacles in answering mathematical proof questions are the students have difficulty in writing mathematical symbolic and inability in mathematical proof. But after being given didactic anticipation by mathematical communication, the ability of students to answer mathematical proof questions has increased. Thus, mathematical communication can be used as an alternative to overcome obstacles or difficulties for students in solving mathematical proof problems.

Keywords: Mathematics Communication; Mathematical Proof; Square and Triangle; Learning Media

1. INTRODUCTION

Experts at the National Council of Teacher of Mathematics (NCTM) include mathematical evidence as one of the components of a standard mathematical process in schools (NCTM, 2000). The inclusion of mathematical evidence into the components of the mathematical process of the standard implies that the topic must be mastered by students. This obligation also has implications for teachers and prospective mathematics teachers to master it in its entirety. The topic of mathematical proof is often a subject of con-versation among experts, because this topic tends to be difficult to learn by students whose mathematical achievements are on average down. Some foreign studies have found that there are still many students who experience difficulties in proving (Ozdemir & Ovez, 2012; Guler, 2016; and Selden & Selden, 2003). This difficulty is also often found by researchers when teaching undergraduate students in real analysis introduction.

Experiences at the Educational Personnel Education In-stitution, Mathematics Education Program Study Program Institute of Teacher Training and Education Pontianak shows that students often have difficulty in proving propositions (theorems in mathematics) (Hodiyanto, 2017). This experience also happened in other regions, this was revealed from the study of Maya and Sumarmo (2014) and Andri (2013). Students generally experience difficulties when planning, implementing plans, and checking the validity of proof of a mathematical proposition. The reason, among others, is hampered from recognizing important information in a mathematical proposition. As a result they: (1) fail to identify what is known and what will be proven; (2) exchanging what will be proven by what is evidence (Weber, 2003; Recio and Godino, 2001). These obstacles occur repeatedly in every mathematics course.

Other things that are predicted to be the main cause of the recurrence of obstacles, namely the instructor uses a formal approach as adopted by structuralist ideas. Mathematics is delivered in the order of lecture methods that have been used so far in the following order: (1) theory / definition / theorem; (2) examples are given; (3) given problem training (Soejadi, 2000). The use of structuralist understanding is not without reason, established students in deductive thinking because they are considered to have existed in the stage of formal thinking so that the order of mathematical presentation as such is justified theoretically. However, such theoretical justi-fication seems to be weakened by the recurring barriers of students who are capable of a downward average in learning proof of mathematics when taught with structuralist ideas. Therefore, it is necessary to find an alternative. The alternative offered through this research is to develop students' abilities in proof through mathematical communication. Some considerations for choosing mathematical commu-nication as an alternative to develop students' abilities in proof. First, mathematical communication is a way of sharing ideas and clarifying mathematical learning understandings. Second, in mathematical communication, ideas come from problem solving processes become objects of reflection, refinement, discussion, and the process of changing ideas if needed (Suryadi, 2013; NCTM, 2000). Third, when students are pro-voked to solve problems, they will have the opportunity to think and try to solve them. Fourth, the obstacles faced by students include: solving problems, using varied ideas, and different solutions will be potential resources to encourage them to share, compare, justify, explain, or discuss problems. Fifth, interactions between students during class activities provide space for the development of their mathematical abil-ities including conceptual understanding and procedural knowledge (Takahashi, 2006). Sixth, interactions between students where mathematical ideas explored from various perspectives can help them to deepen understanding, and develop their ability to communicate, explain, justify, and discuss mathematical ideas. Seventh, in proving students have mathematical dispositions that tend to be low.

Based on the description that has been stated, it is deemed necessary to examine mathematical communication activities as an alternative to overcome the barriers to learning pro-spective teacher students in mathematical proof. Therefore, the research title chosen was "Mathematical Communication as an Alternative to Overcoming Obstacles to Student Teacher Candidates in Proof at IKIP PGRI Pontianak". Based on the background, the general problem of this study is "How do you overcome the difficulties of prospective teacher students in mathematical proof of students in real analysis subjects with mathematical communication? "

2. RESEARCH METHOD

Qualitative research methods are research methods that are based on postpositivism philosophy, used to examine natural object conditions, (as opposed to experiments) where re-searchers are key instruments, purposive or snowball sampling, data collection techniques using triangulation (combined), data analysis is inductive/qualitative, and the results of qualitative research emphasize more meaning than generali-zation (Sugiyono, 2013: 15). Qualitative researchers in col-lecting data occur interactions between data researchers and data sources. In this interaction both researchers and data sources have different backgrounds, views, beliefs, values, interests, and perceptions, so that in collecting data, analyzing, and making reports will be bound by their respective values (Sugiyono, 2013: 21).

This study aims to obtain an overview of what are the barriers to learning experienced by students in proof of real analysis subjects before and after mathematical communica-tion is given, then the form of research used in this study is the form of Case Study. Arikunto said "case studies are a form of research that focuses attention on an intensive, detailed and in-depth case of an organization, institution, or certain symptoms". Ardianto (Arifianto, 2016: 8) says "case studies are approaches in writing that examine each case intensively, deeply, comprehensively and consistently with the concept". The subjects used in this study consisted of two under-graduate students in Mathematics education program IKIP PGRI Pontianak who had been taught real analysis introduc-tions and have the lowest value in answering mathematical proof questions. The data collection tool used was tests and interview. Tests are used to determine student obstacles in answering mathematical proof questions while interviews are used in the application of didactic anticipation in mathemat-ical communication. Data analysis techniques used was descriptions: data reduction, data display and conclusion

3. RESULTS AND DISCUSSION

3.1 Results before Anticipation of Didactics

In this study, after the researcher gave the pretest, the researcher chose two students with the lowest mathematical proof ability. The following will explain the results of pretests before being given didactic anticipation, mathematical communication. In addition, students who were given this pretest were students who had received an introduction of real analysis.

1. **Subject Answer One**
   Answer No. 1

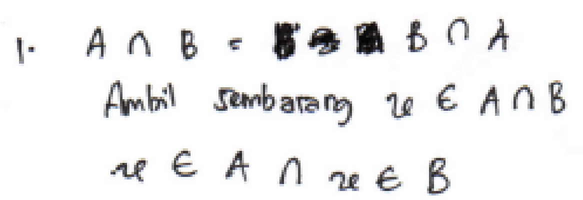


Figure 1. Subject Answer one No. 1

In question number 1, students are asked to prove the commutative theorem of the set A∩B = B∩A. From the answer, subject 1 has not been able to prove and use the definition of the equal set that A = B if only A⊆ B and B ⊆ A. Subject 1 is also still wrong in defining intersection of two sets. $A \cap B = \{x | x \in A \wedge x \in A\}$ not A ∩ B = {x | x∈ A ∩ x ∈ B} as the answer to the subject 1.

Answer No. 2

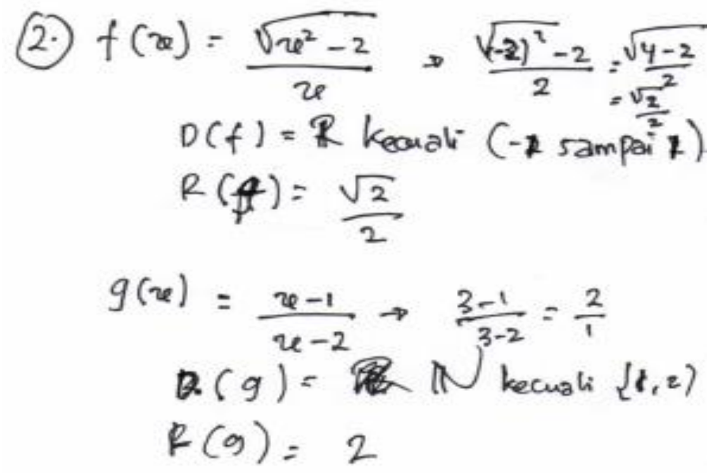


Figure 2. Subject Answer one No. 2

In question number 2, students are asked to determine the domain and range of known functions with real number universes. The function given is in the form of *f (x)* = $\sqrt{x^2-2}$ / x and gx) = (x-1)/(x-2) then students are asked to look for domain (D) and range (R) of *f (x)* and *g (x)*. Subject 1 is wrong in determining D (f) or the domain of *f (x)* and R *(f)* or the range of *f (x)*. Subject 1 is also still wrong in determining D (g) and R (g). The truth are $D(f) = \mathbb{R} \setminus [-\sqrt{2}, \sqrt{2}]$, $R(f) = \mathbb{R}$ and $D(g) = \mathbb{R} \setminus [2]$, $R(f) = \mathbb{R}$.

Answer No. 3

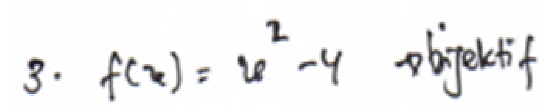


Figure 3. Subject Answer one No. 3

Question 3 is "What is the function *f (x) = x²- 4* and *g (x) = x-3*, the bijective function? If yes, it is proven and if you don't give a denominator! " Subject 1 has not been able to answer the question. From the question *f (x) = x²-4* is not a commodity because it does not fulfill the injective function and *g (x) = x-3* is bijective function.

2. Subject Answer Two

Answer No. 1

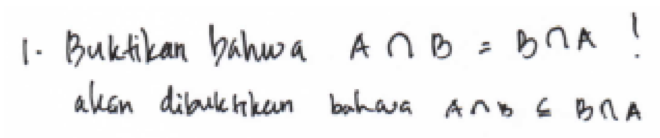


Figure 5. Subject Answer Two No. 1

Based on the answer number 1, subject 2 has begun to use the definition of the same two sets but has not been able to continue the definition of the two sets and continue in proof.

Answer No. 2

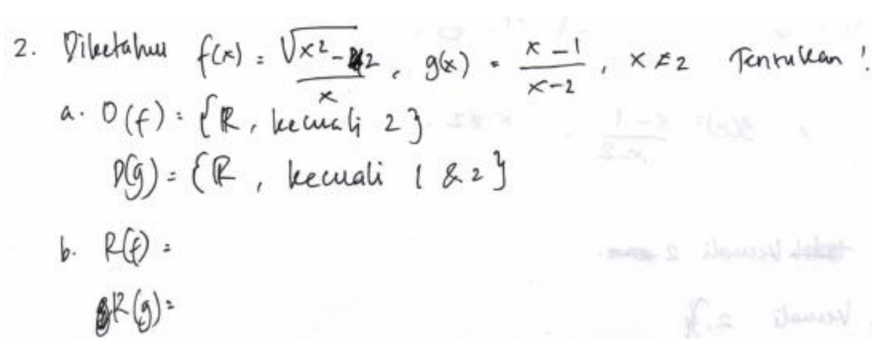


Figure 6. Subject Answer Two No. 2

Based on the answer number 2, subject 2 is wrong in answering the question. In addition, subject 2 is also wrong in writing a set and element of sets where R is included in a set element even though R is a set of real numbers. Correct writing of the answer to subject 2, D (f) = { ℝ except 2} is D (f) = ℝ/ {2} even though this answer is wrong.

Answer No. 3

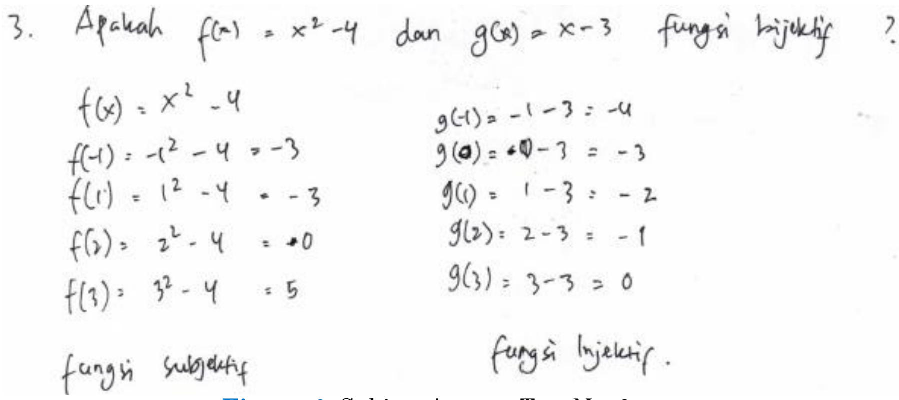


Figure 6. Subject Answer Two No. 3

Based on the answer number 3, subject 2 is still wrong in proof. To prove that the function is a bijective function, one must show that the function is injective and surjective. Other than that, in the proof does not give an example in the domain area such as the answer of the subject 2. But it must be general and certainly based on the definition of the function of injective and surjective. Based on the answers from subject 1 and subject 2, in general it can be concluded that subject 1 and subject 2 students to answer real introductory analysis questions are inability even though they have been taught before.

3.2 Results after Anticipation of Didactics

Student answers before being given the didactic anticipation are still relatively low. Didactic anticipation in this study in mathematical communication between students and lecturers with students. The following will be analyzed student answers after given didactic anticipation.

1. **Subject Answer One**
   Answer No. 1

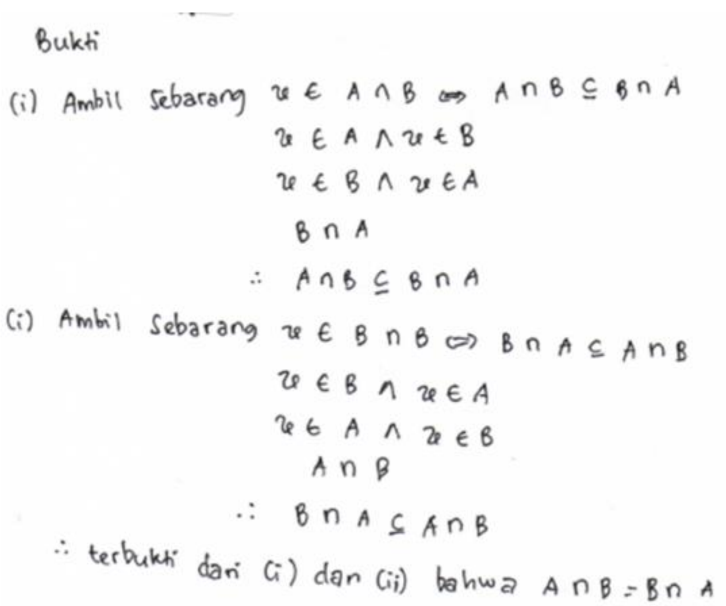
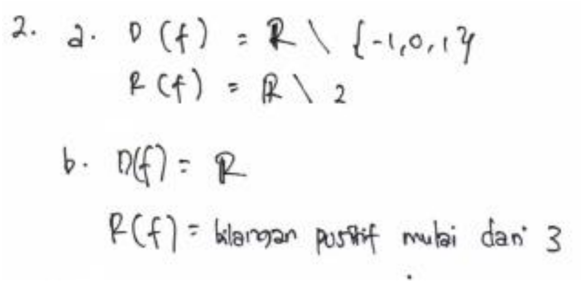


Figure 7. Subject Answer One No. 1

Subject answer 1 after didactic anticipation changes, it means that the answer to subject 1 is correct even though there are some that are less. It should be before doing proof, starting with the definition of the same set. This means that before starting the proof above, subject 2 should start with showing that $A \cap B \subseteq B \cap A$ and $B \cap A \subseteq A \cap B$.

Answer No. 2

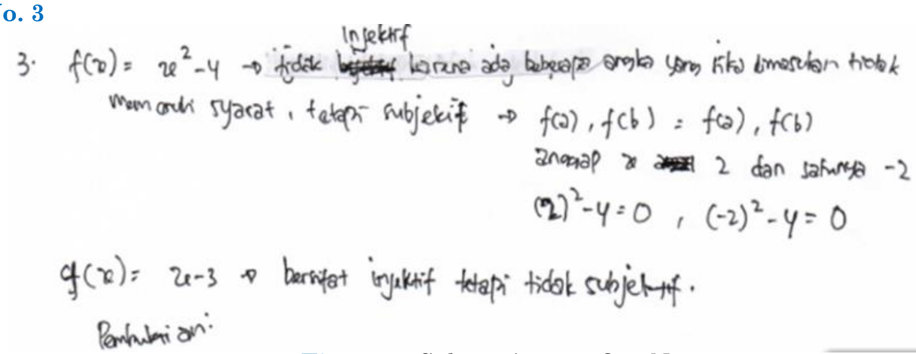

2. a. $D(f) = \mathbb{R} \setminus \{-1, 0, 1\}$
$R(f) = \mathbb{R} \setminus 2$

b. $D(f) = \mathbb{R}$
$R(f)$ = bilangan positif mulai dari 3

Figure 8. Subject Answer One No. 2

Answer subject 1 after didactic anticipation changes. Answer D (f) is almost correct but it is wrong to determine the exception that should be $D(f) = \mathbb{R} \setminus \left[-\sqrt{2}, \sqrt{2}\right]$ even though the others are still wrong. But if compared to before being given didactic anticipation, subject 1 is wrong to answer and even wrong in writing.

Answer No. 3


3. $f(x) = x^2 - 4 \rightarrow$ tidak injektif karena ada beberapa angka yang kita masukkan tidak memenuhi syarat, tetapi subjektif $\rightarrow f(a), f(b) = f(a), f(b)$
anggap x 2 dan satunya -2
$(2)^2 - 4 = 0$, $(-2)^2 - 4 = 0$

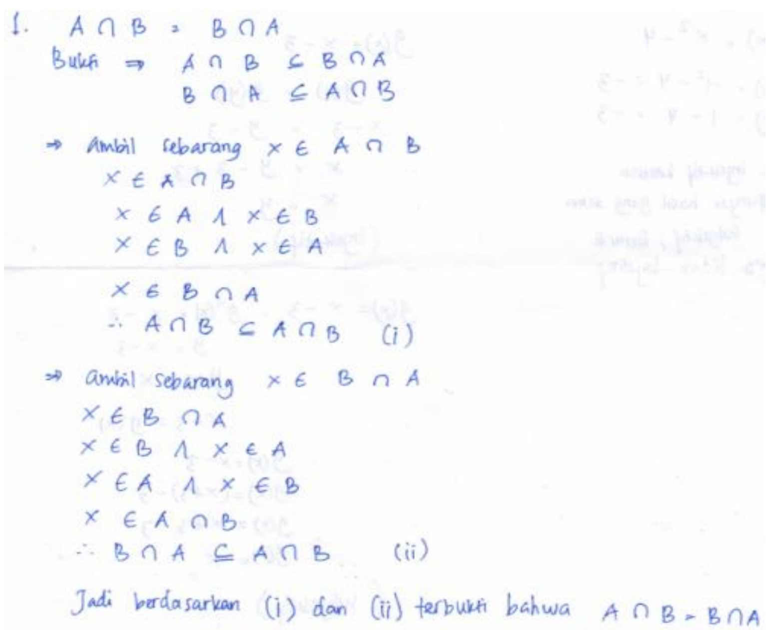
$g(x) = x - 3 \rightarrow$ bersifat injektif tetapi tidak subjektif.
Pembuktian:

Figure 9. Subject Answer One No. 3

The answer to subject 1 after being given anticipation of didactic changes. Subject 1 answers correctly that *f (x) =* x^2*-4* is not a bijective function because it does not fulfill injective function and subject 1 is correct in giving a denominator by specifying domain $a \neq b$ which is $-2 \neq 2$ but $f(2) = f(-2)$. But subject 1 is still wrong in answering $g(x) = x - 3$ is not a bijective function even though *g (x)* is included as a bijective function meaning *g (x)* is an injective function and surjective function.

2. Subject Answer two

Answer No. 1


1. $A \cap B = B \cap A$
Bukti $\Rightarrow$ $A \cap B \subseteq B \cap A$
$B \cap A \subseteq A \cap B$

$\Rightarrow$ Ambil sebarang $x \in A \cap B$
$x \in A \cap B$
$x \in A \wedge x \in B$
$x \in B \wedge x \in A$
$x \in B \cap A$
$\therefore A \cap B \subseteq A \cap B$ (i)

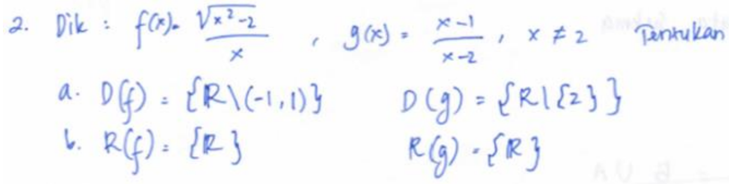
$\Rightarrow$ ambil sebarang $x \in B \cap A$
$x \in B \cap A$
$x \in B \wedge x \in A$
$x \in A \wedge x \in B$
$x \in A \cap B$
$\therefore B \cap A \subseteq A \cap B$ (ii)

Jadi berdasarkan (i) dan (ii) terbukti bahwa $A \cap B = B \cap A$

Figure 10. Subject Answer Two No. 1

Subject answer 2 after didactic anticipation changes, it means that the answer to subject 2 is correct in proving A∩B = B∩A. But it would be better if each step is given information such as $x \in A \wedge x \in B$ is the definition of intersections.

Answer No. 2


2. Dik : $f(x) = \frac{\sqrt{x^2-2}}{x}$, $g(x) = \frac{x-1}{x-2}$, $x \neq 2$ Tentukan

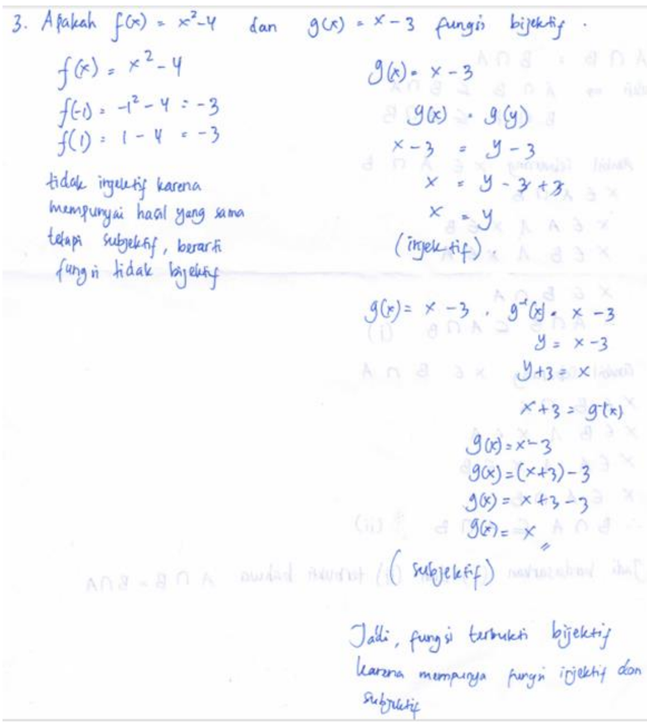
a. $D(f) = \{R \setminus (-1,1)\}$ $D(g) = \{R \setminus \{2\}\}$

b. $R(f) = \{R\}$ $R(g) = \{R\}$

Figure 11. Subject Answer Two No. 2

Subject answer 2 after anticipation of didactics is a few changes before being given didactic participation. Subject 2 is still wrong in writing the set and members of the set. The answers D (g) and R (g) are almost correct but subject 2 is wrong in writing the answer that should be D (f) = R \ {2} and R (f) = R.

Answer No. 3


3. Apakah $f(x) = x^2-4$ dan $g(x) = x-3$ fungsi bijektif .

$f(x) = x^2-4$

$f(-1) = -1^2-4 = -3$

$f(1) = 1-4 = -3$

tidak injektif karena mempunyai hasil yang sama tetapi subjektif, berarti fungsi tidak bijektif

$g(x) = x-3$

$g(x) = g(y)$

$x-3 = y-3$

$x = y-3+3$

$x = y$

(injektif)

$g(x) = x-3$, $g^{-1}(x) = x-3$

$y = x-3$

$y+3 = x$

$x+3 = g^{-1}(x)$

$g(x) = x-3$

$g(x) = (x+3)-3$

$g(x) = x+3-3$

$g(x) = x$

(subjektif)

Jadi, fungsi terbukti bijektif karena mempunyai fungsi injektif dan subjektif

Figure 12. Subject Answer Two No. 3

Subject answer 2 after being given anticipation of didactic changes. Subject 1 answers correctly that $f(x) = x^2-4$ is not a bijective function because it does not fulfill injective function and subject 2 is correct in giving a denial by specifying a domain $a \neq b$ which is $-1 \neq 1$ but $f(1) = f(-1)$. In addition, subject 2 also almost correctly proves that $g(x) = x-3$ is a wise function. But subject 2 in the answer or prove that g (x) is bijective function with anstructured answer. As in proving $g(x)$ injective function, subject 2 goes straight to the definition of injective function even though it should start with taking any x and y member real numbers. Likewise in proof of $g(x)$ surjective function. As if taken by any member B, there are members A such that $g(a) = b$ and select $a = b + 3$.

3.3 Discussion

Based on the results of the research before giving didactic anticipation in mathematical communication, subject 1 and subject 2 still had difficulty in answering the questions of real analysis introduction in the form of mathematical proof ability. From the answers to subject 1 and subject 2, they were wrong in answering and even wrong in writing the answers. Even before being given this pretest, students had been taught the material that was used as the pretest. But the results are still not optimal. This means that the learning process carried out is not optimal so that researchers try to anticipate didactic in mathematical communication. Some foreign studies have found that there are still many students who experience difficulties in proving (Ozdemir & Ovez, 2012; Guler, 2016; and Selden & Selden, 2003). Therefore, the researcher tries to anticipate didactic in the form of material communication as an alternative in overcoming student barriers in proof. After the didactic anticipation in the form of mathematical communication is carried out, the student test results show changes or improvement in students in answering the introductory questions of real analysis. Subject 1 students who had not been able to answer questions 1 and 3 before but after anticipation of didactic subject 1 were able to answer the question even though there were few errors. Likewise for subject 2, who had previously not been able to prove the question given but after being given didactic anticipation, she can solve it. This success is certainly due to obstacles faced by students, among others: solving problems, using varied ideas, and different solutions will be potential resources to encourage them to share, compare, justify, explain, or discuss problems. In addition, interactions between students during class activities provide space for the development of their mathematical abilities including conceptual understanding and procedural knowledge (Takahashi, 2006). Interactions between students where mathematical ideas are explored from various perspectives can help them to deepen understanding, and develop their ability to communicate, explain, justify, and discuss mathematical ideas.

4. CONCLUSION

Based on the results of research, study of theory and discussion, it could be concluded in this study that the student obstacles in answering mathematical proof questions are the students have difficulty in writing mathematical symbolic and inability in mathematical proof. But after being given didactic anticipation by mathematical communication, the ability of students to answer mathematical proof questions has increased. Thus, mathematical communication can be used as an alternative to overcome obstacles or difficulties for students in solving mathematical proof problems. Based on the results of the study, the researchers put forward some suggestions that could be used to develop the next research and also become input for teachers and even other researchers. The suggestions are as follows: 1). What student obstacles in solving mathematical proof. 2). Giving questions related to introduction of real analysis is given more in various so that obstacles or student difficulties are more revealed. 3). Mathematical communication can be an alternative to increase mathematical proof, but it would be better if it is accompanied by other innovative models or learning.

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AUTHOR'S CONTRIBUTIONS

The authors discussed the results and contributed to from the start to final manuscript.

CONFLICT OF INTEREST

There are no conflicts of interest declared by the authors.

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