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Research Article

Students' Cognitive Processes in Understanding the Concept of Limit Function

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ABSTRACT

The purpose of this study was to determine the description of students' cognitive processes in understanding the concept of limit function. This is a qualitative descriptive study. We determine student characteristics in understanding the concept of limits. Students' physical and mental activities are analyzed based on actions, processes, objects and schemes in the form of genetic decomposition. It is recorded in audio-visual form. The subjects of this study were 18 mathematics education students in Bengkulu. Subjects were selected based on the task they performed in understanding the limit of function. Data collection was carried out using paper and pencil and task-based interviews. Its task is to ask students to prove the truth about the limit function. The data were analyzed qualitatively and the comparative technique was constant. The results of this study were that only a small proportion of students were able to carry out a complete cognitive process on the concept of limits. It was only 2%, but he was able to prove the truth of a statement about limits. The subject is able to carry out coherent cognitive process activities. The conclusion is that there is a minority of students who are able to build linkages between action-process processes, objects and other schemes, so that a mature scheme is formed. Thus it is suggested that lecturers choose a learning approach that is close to students' cognitive activities..

Keywords: Cognitive Processes; Understanding; Limit Function

1. INTRODUCTION

Geometry learning is one of the mandatory materials for Function limit is one of the content that is difficult for students to understand. The basic concept of limit function involving epsilon-delta is often a hindrance, but teachers are in a hurry to convey this concept. Most of the teachers immediately gave examples, to make it easier for them to complete their teaching materials. This results in students understanding more examples than conceptually understanding the limit function (Liang, 2016). Therefore, teachers in teaching the topic of limits must first develop the concept before applying the formula. Many students experience misconceptions about limit functions (Denbel, 2014).

Limit is a basic concept of calculus which is closely related to other concepts. It is continuity, differentiation, and integral (N. Cetin, 2009). The results of his research show that students are able to calculate the limit value by applying standard procedures but cannot use the concept of limits in solving related problems. In much of our experience teaching calculus, students have difficulty understanding the concept of limits. The idea of the limit function is a basic concept for understanding calculus (I. Cetin, 2009). Understanding the concepts of derivatives and integrals requires a good understanding of the limit function.

There are students' cognitive processes regarding the application of function limits, function continuity and derivatives in terms of Action-Process-Object-Scheme (APOS Theory) (Baker, Cooley, Trigueros, & Trigueros, 2000). They stated that the students had difficulty reaching the schema about the infinite limit on the first derivative function at $x = 0$. The difficulties experienced by the students were in the action-interiorization-process-encapsulation-thematization-schema object. It is about coordinating some of the principles of first and second derivatives, limits and derivatives, continuity, domains, and ranges. As a result, the students had difficulty regarding the vertical asymptote at $x = 0$, had difficulty coordinating the analytic conditions with the graphical properties of the function, and the students were unable to develop mature schemas. In another study, it was stated that the error in the concept of limit functions resulted in students' obstacles in understanding the concepts and principles of derivatives (Wahyu Widada & Herawaty, 2017).

Students are active information processors. They are able to represent any information according to the level of knowledge they have (D. Herawaty, Widada, Novita, Waroka, & Lubis, 2018)(Wahyu Widada, Herawaty, Nugroho, & Anggoro, 2019a). Teachers can use it as a means of achieving learning goals. Through the process of student cognition, teachers should not only emphasize student behavior that is easily observed, but also emphasize what is happening in the minds of students. Students' cognitive processes regarding limits can be analyzed through their genetic decomposition. The results of the study describe the genetic decomposition of how students learn the concept of limits (I. Cetin, 2009). It is the act of evaluating f at a point x which is considered to be close to, or even equal to, a. The action evaluates the function f at several points, each successive point closer to a than the previous point. The construction of a coordinated scheme that includes, interiors these actions to construct a process domain where x approaches a. Construction of a span process where y approaches L. Also, the coordination (a), (b) through f. Students 'understanding of the concept of limits requires students' abilities about algebraic, verbal and graphic representations. Verbal, graphic and algebraic representations can improve the development of students' interpretation skills of various representations of function (Karatas, Guven, & Cekmez, 2011).

Students' cognitive processes can be analyzed through mental and physical activities related to schema behavior. Schema is a guide in organizing the information entered in the memory system in a collection of knowledge (Dewi Herawaty, Widada, Herdian, & Nugroho, 2020). Schematic behavior is a pattern of interconnected ideas that are clearly linked in a person's mind. The pattern of these ideas can be seen through a person's mental constructions that are made to achieve and understand mathematical concepts and principles. Understanding of mathematical concepts is the result of construction or reconstruction of mathematical objects. The construction or reconstruction is carried out through activities in the form of mathematical actions, processes, objects organized in a scheme to solve a problem. This can be analyzed through a genetic decomposition analysis as an operationalization of the APOS theory (Action, Processes, Object, and Schema)(Dubinsky, 2000)(Dubinsky & McDonald, 2000)(Dubinsky, Weller, McDonald, & Anne, 2013). APOS theory is a constructivist theory of how it is possible to achieve / learn a mathematical concept or principle, which can be used as an elaboration of mental constructions from actions, processes, objects and schemes. (Dubinsky et al., 2013)(Baker, Cooley, & Trigueros, 2000). It's a genetic decomposition (Wahyu Widada, Efendi, Herawaty, & Nugroho, 2020) (Inglis, 2012).

Genetic decomposition is a structured collection of mental activities performed by a person to describe how mathematical concepts / principles can be developed in his mind. Genetic decomposition analysis is an analysis of a genetic decomposition based on the activity of actions, processes, objects and schemes that a person carries out in understanding a mathematical object or solving a problem (Wahyu Widada et al., 2020). Students in learning see limits as unreachable, they see limits as estimates, see limits as limits, see limits as dynamic processes and not as static objects (Denbel, 2014). Many factors influence the understanding of function limits. These factors include low conceptual understanding of functional limits, overgeneralization, inconsistent cognitive structures, incoherent reasoning and factual errors (such as symbolic notation). Students' thinking strategies depend on concept images rather than concept definitions (Sebsibe & Feza, 2019). The results showed a relationship between sources of belief and understandin g of limits. Students with external sources of belief made mistakes in defining limits, and had more misconceptions about limits (Szydlik, 2000). Therefore, it is necessary to explore how the students' cognitive processes are. That is how to describe the cognitive processes of students in understanding the concept of limit function.

2. RESEARCH METHOD

This research is part of development research. This stage is a qualitative descriptive study. We determine student characteristics in understanding the concept of limits. Students' physical and mental activities are analyzed based on actions, processes, objects and schemes in the form of genetic decomposition. It is recorded in audio-visual form. The subjects of this study were 18 mathematics education students in Bengkulu. Subjects were selected based on the task they performed in understanding the limit of function. The subject of the task-based analysis he worked on. Data collection was carried out using paper and pencil and task-based interviews. Its task is to ask students to prove the truth about the limit function. The task described in this paper is that students are asked to prove that $\lim_{x\to 3}(2x-1)=5.$ The data were analyzed qualitatively and the comparative technique was constant.

3. RESULTS AND DISCUSSION

In this pandemic, we present calculus subject matter using the youtube media. We have done this to provide an understanding of the basic concepts of limit functions. Students can study anytime and anywhere. They can repeat it through these youtube videos.

After one week students were asked to understand the concept of limits, they were asked to prove that $\lim_{x\to 3}(2x-1)=5.$ Students were interviewed online via video-call. Interview transcriptions were analyzed for genetic decomposition. It is a series of activities of action (A), process (P), object (O), and schema (S) or a combination thereof. Based on the genetic decomposition, students' cognitive processes in understanding the concept of limits can be seen in **Figure 1**.

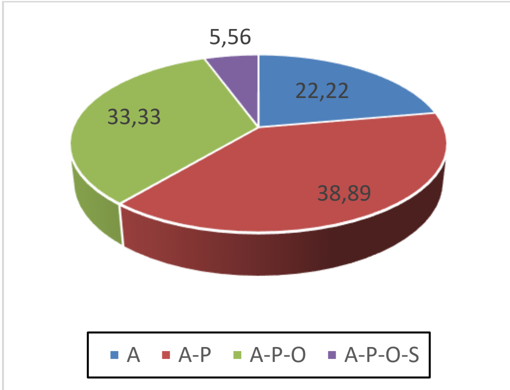

Figure 1. Presentation of students' cognitive processes in understanding the concept of limits

Based on Figure 1, there are 22,22% of students who are only able to take actions, 38,89% of students can interiorize actions so that they become a process. The figure also shows that there are 33,33% of students who can encapsulate actions, the process becomes an object regarding the concept of limits, and only 5,56% of students can complete cognitive activities from actions, processes, objects so that a scheme of concepts is reached.

Based on further investigation, students who can complete cognitive activities from actions, processes, objects so that a scheme of limits is built are those who learn contextually based on students' cognitive activity. The lecturer first analyzes the student's needs. That is through searching about the previous schema that students have as a basis for learning. These processes are interiosrization, encapulation, and thematization (Dubinsky & Mcdonald, 2001). Interiorization is an activity change from a procedural activity to be able to re-perform the activity in imagining several meanings that affect the resulting conditions (a form of action in the process) (Wahyu Widada, 2006). As an indication, it is doing an activity change from an action to an activity that is carried out internally (imagined in the mind). If all actions can be placed in the mind of the individual or can be imagined without carrying out the whole of certain stages, then action after being interiorized becomes a process. As an indication, it is carrying out internal activities but not necessarily directly from external stimuli (not all stages are done explicitly). Encapsulation is a process performed on objects. As an indication is to carry out a mental transformation (in the form of cognitive coordination) of a process on a cognitive object (Wahyu Widada, 2007). If the process itself is transformed by several actions, then the encapsulation matches an object. As an indication is the formation of a single entity from the encapsulation of several processes. Thematicization is a construction that links separate actions, processes, or objects to a particular object to produce a schema. As an indication is the formation of a total entity from different objects that are constructed through cognitive coordination (Wahyu Widada, 2011). Furthermore, the students' cognitive processes (Ty) as stated in paper-and-pencil can be seen in.

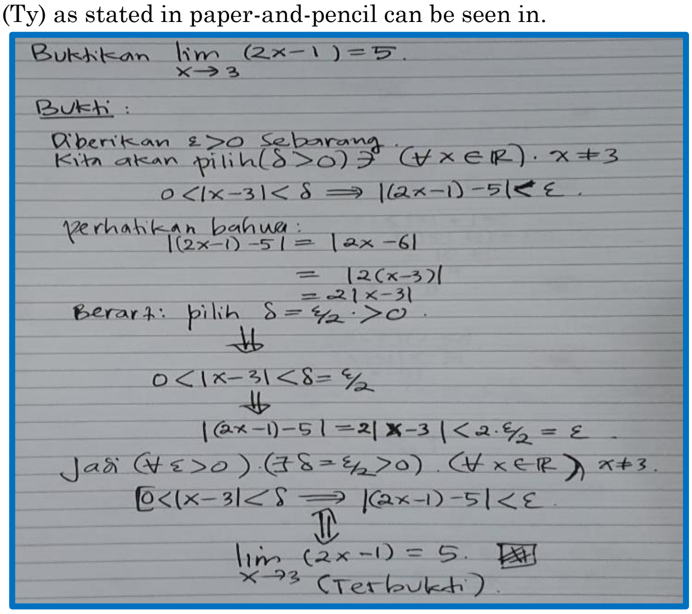

Figure 2. Students' complete cognitive processes (A-P-O-S)

Based on the genetic decomposition analysis, Subject Ty was able to perform the following cognitive processes.

Q : How do you show that the limit of 2x-1 for x approaches 3 is 5?

Ty : I recall the statement that for every positive epsilon there is such a positive delta that for all real numbers x holds: if the absolute value of (x-3) is more than 0 and less than delta, then the absolute value (2x-1) - 5 is less from epsilon.

Q : Fine… you show me now.

Ty : I've tried to do it on paper.

Q : Please describe it.

Ty : Ok… the absolute price (2x-1) - 5 is the same as 2 times the multak price (x-3).

Q : ……

Ty : For example given positive epsilon, then I choose delta to be half of epsilon. If the absolute value of (x-3) is more than 0 and less than half the epsilon, then the absolute value (2x-1) - 5 is less than epsilon. (see Figure 2)

Q : Ok fine ...

Ty : It is thus proven that the limit of 2x-1 for x approaches 3 is 5.

The interview snippet with Ty illustrates that Ty performs a coherent cognitive processing activity. The actions are interrogated into a delta-seeking process. Ty finds delta is half of epsilon. Ty encapsulates it into a statement that "If the absolute value of (x-3) is more than 0 and less than half the epsilon, then the absolute value (2x-1) - 5 is less than epsilon." He produced a mature schema that the proving process is achieved by concluding that the limit of 2x-1 for x approaches 3 is 5. Thus, Ty is able to establish linkages between process-process actions, objects and other schemes. It is a mature scheme.

When analyzed, two statements that are true are equivalent. Based on (Bartle, 2000) analyzed two equivalent statements. Statement 1: Let $A \subseteq R$, f: $A \to R$, $c \in R$ cluster point A. Lim $x \to c f(x) = L$ if and only if for every V neighborhood from L there is a U neighborhood c such that for every x, prevail, if $x \neq c$, $x \varepsilon A \cap U$, then $f(x) \in V$. Statement 2: Let l $A \subseteq R$, f: $A \to R$, $c \in R$ cluster point A. Lim $x \to c f(x) = L$ if and only if for each $\varepsilon > 0$, exist $\delta > 0$, such that for every x prevail, if $0 < |x-c| < \delta$ and $x \in A$, then $|f(x)-L| < \varepsilon$. Definition 1: Let $a \in R$. For every $\varepsilon > 0$, ε-neighborhood a is the set $V\varepsilon(a) = \{x \in R \mid x-a \mid < \varepsilon\}$. A neighborhood a is any set containing ε-neighborhood a, for $\varepsilon < 0$. Definition 2: $r \varepsilon R$ is cluster point $A \subseteq R$, if and only if every ε-neighborhood r contains at least one point of A other than r.

Based on Definition 1 and Definition 2, Statement 1 can be shown to be equivalent to Statement 2. This means it must be shown that "If Statement 1 then Statement 2, and Statement 2 then Statement 1. Let Lim $x \to c f(x) = L$, $\varepsilon > 0$, from Definition 1 and Statement 1, then for arbitrary $V\varepsilon(L) = (L-\varepsilon, L+\varepsilon)$ ε-neighborhood L there exits U neighborhood c, if $x \neq c$, $x \in A \cap U$, then $f(x) \in V\varepsilon(L)$. $f(x) \in V\varepsilon(L) \Rightarrow \mid f(x)-L \mid < \varepsilon$. Then because U is neighborhood c, means there is $\delta > 0$, such that $(c-\delta, c+\delta) \subseteq U$ (Definition 1). It means $0 < |x-c| < \delta$, $x \in A \Rightarrow x \in A \cap U$ and $x \neq c$. Thus every x is valid, if $0 < \mid x-c \mid < \delta$ and $x \in A$, then $\mid f(x)-L \mid < \varepsilon$. Let Limx-cf(x) = L. and V is neiighborhood of L, then $V\varepsilon(L) = (L-\varepsilon, L+\varepsilon) \subseteq V$, for $\varepsilon > 0$. Based on Statement 2, there is $\delta > 0$ such that, for every x prevail, if $0 < \mid x-c \mid < \delta$ and $x \in A$, then $\mid f(x)-L \mid < \varepsilon$, it means $f(x) \in V\varepsilon(L)$. Let $U = (c-\delta, c+\delta)$, for $0 < \mid x-c \mid < \delta$ and $x \in A$, mean $x \in A \cap U$. Thus it can be concluded that for every x, if $x \neq c$, $x \in A \cap U$, then $f(x) \in V\varepsilon(L) \subseteq V$ (analogous to some of these writings, (W Widada et al., 2020)(Wahyu Widada et al., 2020)(Dewi Herawaty et al., 2020)). It is a formal understanding, which is also found in many textbooks that develop students' understanding of the formal definition of limits. Understanding these students causes learning mechanically to apply counting procedures and techniques rather than conceptual understanding (Liang, 2016). Therefore we need a learning approach that is close to the student cognitive process. It is done through understanding the needs of students' thought processes. Students can build their own through the process of interiorization and encapsulation. It is a constructivist approach (Clark et al., 1997) (Tossavainen, n.d.) (Andriani et al., 2020). Therefore, the genetic decomposition of students after learning can reach a mature scheme (Wahyu Widada, Herawaty, Nugroho, & Anggoro, 2019b) (D Herawaty, Sarwoedi, Marinka, Febriani, & Wirne, 2019) (W. Widada, Herawaty, Nugroho, & Anggoro, 2019). Thus, competencies regarding function limits can be achieved well through learning that is close to the minds of students. It is learning based on constructivist theory in accordance with the flow of action-process-object-schema.

4. CONCLUSION

We found that only a few students were able to understand the concepts and principles of limits conceptually. That's only 2%. He is able to build linkages between action-process processes, objects and other schemes, so as to form a mature scheme. These students are those who take calculus learning through a learning approach that is close to the cognitive activities of students.

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AUTHOR'S CONTRIBUTIONS

All authors discussed the results and contributed to from the start to final manuscript.

CONFLICT OF INTEREST

There are no conflicts of interest declared by the authors.

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