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Research Article

The development of two-variable function derivative learning using GeoGebra

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ABSTRACT

Mathematics learning model using GeoGebra software has become a handy tool in teaching mathematical concepts and applications. Students can conduct experiments and explore mathematical concepts in an interactive and fun way with GeoGebra software. Using GeoGebra in learning mathematics of two-variable functions can increase students' under-standing on the topic discussed, strengthen mathematical computational and visualization skills, and motivate them to know more and better complex mathematical concepts. From this perspective, this study aims to know the impact of using GeoGebra on students' understanding of the derivative of a function of two variables. Furthermore, this study was con-ducted on two groups of students, namely the control group (not using GeoGebra) of 53 students and the experimental group (using GeoGebra) consisting of 50 students. Based on these results descriptively students in the experimental group have higher scores on the assessment of skills in determining partial derivatives and their application. Statistically, there is a difference between the experimental and control groups in understanding the function of the derivative of the two variables.

Keywords: Mathematics Learning; Derivative; GeoGebra Software; Statistically; Computer software

1. INTRODUCTION

Computer software, in particular mathematics software, nowadays has an increasingly greater role in the learning process that applies modeling analysis and complex mathematical calculations. Utilization software for Mathematics can be an integral part of learning renewal. Software GeoGebra is designed to help students improve their understanding of mathematics (Hohenwarter, no date) (Alkhateeb and Al-Duwairi, 2019). At the Technical University of Liberec, the topic of learning geometry is easier to understand with the help of the Geogebra software (Pirklová and Bímová, 2019). Through the help of this software, students are more active and faster in understanding mathematical concepts (Ya'Acob, Mohamed, and Ariffin, 2016). This software provides three approaches to determining mathematical solutions, namely: dynamic, numerical, and algebraic. So that the use of this technology is an easy key to understand mathematical problems (Hernández, Perdomo-Díaz and Camacho-Machín, 2020) (Hohenwarter and Fuchs, 2005).

GeoGebra is an open-source (free) dynamic mathematics software, which interactively combines 2D plane geometry, 3D space, and computer algebra system (CAS) by Markus Hohenwarter at the University of Salzburg (2001). Furthermore, at Florida Atlantic University (2006-2008) this software was developed into a combination of geometry, algebra, spreadsheets, graphs, calculus, and statistics in one easy-to-use package so that this software is widely used for learning and solving mathematics (Weinhandl et al., 2020). This software is designed for teaching and learning mathematics so that mathematical concepts are clearer and more easily understood by students. Besides that, it can be used for solving and developing mathematical problems (Ziatdinov and Valles, 2022). The advantage of GeoGebra software compared to other mathematical software such as Maple, Derive, and Matlab is the ability to determine a mathematical solution according to the changes we want.

The use of technology in learning mathematics is an inseparable part of increasing understanding of mathematics. GeoGebra is a means for students to improve their understanding of mathematics more easily (Augusto et al., 2023) (Zengin, Furkan and Kutluca, 2012). Teaching mathematics using GeoGebra in terms of strengthening conceptual knowledge is superior to conventional methods (Armando et al., 2021) (Narh and Sabtiwu, 2022). In learning calculus with

the topic of differential equations grouped into two groups, the score achieved by the experimental group (using GeoGebra) is a higher score compared to the control group (not using GeoGebra) (Latifi et al., 2022). The results of research on understanding calculus topics on derived sub-topics, students' understanding with the help of Geogebra is better than traditional learning models (Yerizon, Fatimah and Tasman, 2021). Furthermore, in research on teaching mathematics using GeoGebra, several researchers found that this software is able to improve understanding of mathematics (Attorps, Björk and Radic, 2016) (Saha, Ayub and Tarmizi, 2010).

With GeoGebra, learning the application of mathematics becomes exploratory, where students can see directly and instantly the linkages between analytic and visual representation of a concept as well as the interrelationships between mathematical concepts (Yatim et al., 2022) (Hall and Chamblee, 2013). Furthermore, the use of GeoGebra in teaching differential equations in the two GeoGebra-assisted groups and the traditional group, shows that the level of mastery of the GeoGebra-assisted group is superior in terms of analytic, visual, and integrated (Latifi, Hattaf and Achtaich, 2021). This shows the importance of this research, which needs to be developed so that it can contribute to the development of more modern mathematics learning. A review of literature studies in the field of teaching mathematics with the help of GeoGebra shows the importance of using technology in improving mathematics learning. Geogebra-assisted mathematics learning generally discusses the calculus of one-variable functions. In this study, the teaching of two-variable function calculus mathematics was developed using GeoGebra with the sub-topic of two-variable function derivatives.

2. RESEARCH METHOD

Sample and Data Collection

This research was conducted in Politeknik Negeri Bandung in 2022-2023. In this research, one hundred and three students were used as the samples which were divided into two groups, namely the control group of fifty-three students and the experimental group of fifty students. The GeoGebra introductory module was developed to allow students to become familiar with the software and explore its features and functions. The experimental group had got the lesson of the derivative function of two variables integrated with GeoGebra, while the control group had got the lesson in a classical construction approach. To measure the similarity and difference of the lesson understood by the students in the two groups, a pre-test was carried out. This research was conducted for two weeks and at the end of the second week a post-test was carried out. The pretest questions were knowledge of calculus of functions of one variable which is a requirement for lectures of calculus of functions of two variables. While the post-test questions were questions that consist of first, and second partial derivatives, chain rules, and extreme values.

Data Analysis

The results of the pre-test were measured by the similarity of the basic abilities of the two groups descriptively and with Independent Samples t-test. Post-test results determined the difference between the two groups descriptively and using independent Samples t-test. If one of the pre-test and post-test data groups is not normal, then a non-parametric approach was applied through the Mann-Whitney test. All data were analyzed with the SPSS program.

Teaching Derivative of a two-variable with GeoGebra (An Example)

The following process illustrates the solution of partial derivatives (chain theorem) using GeoGebra. This problem is part of learning the derivative of a function of two variables. The activity carried out is to determine the derivative of w against t (dw/dt) for $W = xy^2+xy$; $x = \cos(2t)$ dan $y = e^t \sin(3t)$. Then calculate the W derivative for $t = \pi$.

1. Open GeoGebra, in the input type W = xy2 + xy then find the first derivative with respect to x and using Derivative mathematical functions

   $W(x,y) = xy^2 + xy$

   $Wx(x,y) = \text{Derivative}(W,x)$

   $= y^2 + y$

   $Wy(x,y) = \text{Derivative}(W,y)$

   $= 2xy + x$

   After that, students were required to observe the partial derivatives of x by assuming constant y as well as the partial derivatives of y by assuming constant x. To improve understanding, students can explore by changing the forms of the function w.

2. Derive the x and y functions with respect to t respectively. This reduction uses the multiplication and addition derivative theorems, students observed the results of the reduction and to improve understanding, students can change the forms of functions x(t) = P(t) and y(t) = Q(t).

   P(t) = t sin(2t)

   Pt(t) = Derivative (P, t)

   = sin(2t) + 2t cos(2t)

   Q(t) = et cos(3t)

   Qt(t) = Derivative (, t)

   = e^t cos(3t) – 3 e^t sin(3t)

3. Determine the derivative of W with respect to t and calculate for t = π

   Wt(t, x ,y) = Wx(x, y)Pt(t) + Wy(x, y)Qt(t)

   = $(y^2 + y)(\sin(2t) + 2t\cos(2t)) + (2xy + x)(e^t \cos(3t) - 3e^t \sin(3t))$

   f(t) = Wt(t, P, Q)

   = $((e^t \cos(3t))^2+e^t\cos(3t))(\sin(2t)+2t\cos(2t))+((2t\sin(2t)et\cos(3t)+t\sin(2t))(e^t \cos(3t) -3et \sin(3t))$

   f(π) = 3219.2

   In this final solution, students were required to observe the substitution of functions so that the final result was obtained in the form of variable t. Students can do this activity repeatedly with different problems, so that in a short time students can do this activity at least four times with the help of GeoGebra.

3. RESULTS AND DISCUSSION

Pre-test Scores of the Control and Experiment

Based on the results test of the 53 control group, the mean and standard deviation were 1.962 and 0.921, respectively, while the results test of 50 people in the experimental group respectively 1.96 and 1.957 (see **Table 1**) which means that descriptively the two groups have the same basic abilities.

Table 1. Descriptive of the Pre-Test Scores

Controls		Experiments	
Mean	1.962264151	Mean	1.96
Standard Deviation	0.960013304	Standard Deviation	1.399125091
Sample Variance	0.921625544	Sample Variance	1.95755102
Minimum	1	Minimum	0
Maximum	6	Maximum	4
Count	53	Count	50

Based on the results of data normality testing test size 53, for the control group, Asym sig = 0.00 < 0.05 and the experimental group, Asym sig = 0.001 < 0.05 (see **Table 2**), this means that the data of the two groups is not normal, so as to test the hypothesis that the two groups have an average ability that is the same will be carried out through the approach nonparametric with the Mann Withney test obtained Asym sig 0.728 > 0.05 (See **Table 2**) which means that Ho is accepted, meaning that both groups have the same basic abilities.

Table 2. Mann-Whitney Test of the pre-test scores

	Tests of Normality				Test Statistics	
	Groups	Kolmogorov-Smirnov				
		Statistic	df	Sig.	Mann-Whitney U	Asymp. Sig. (2-tailed)
Pre-Test Scores	Controls	0.352	53	0	1274.5	0.728
	Experiments	0.171	50	0.001		

The test for the difference between the Control and Experiment Groups

The post-test results of 53 control group students and 50 experimental group students are described in **Table 4**, the mean and standard deviation of the control group are 47.40 and 10.753 respectively while for the experimental group are 64.24 and 17 respectively. 754. Furthermore, 95% of the post-test scores for the control group were in the interval [44.43 50.36] while the experimental group was in the interval [59, 69.29]. This means that descriptively the experimental group has a higher ability to understand the mathematics of functions of two variables than the control group. Based on the results of testing the normality of the data of the two groups, it was obtained for the control group, Asym sig = 0.00 <0.05 and the experimental group, Asym sig = 0.006 < 0.05 (see **Table 4**), this means that the data of the two groups is not normal so as to test the hypothesis that the two groups have an average of different abilities will be tested the hypothesis through a nonparametric approach with the Mann Withney test obtained Asym sig 0.000 < 0.05 (See **Table 4**) which means that Ho is accepted meaning that the experimental group has better abilities than the control group.

Table 3. Descriptive Post-test scores

Controls		Experiments	
Mean	4.943396226	Mean	6.52
Standard Deviation	1.081578162	Standard Deviation	1.728984602
Sample Variance	1.169811321	Sample Variance	2.989387755
Minimum	3	Minimum	4
Maximum	8	Maximum	10
Count	53	Count	50

Table 4. Mann-Whitney Test of the post-test scores

	Tests of Normality				Test Statistics	
	Group	Kolmogorov-Smirnova				
		Statistic	df	Sig.	Mann-Whitney U	Asymp. Sig. (2-tailed)
Post-Test Scores	Control	0.224	53	.000	636.5	.000

This research resulted in learning the function of two variables using GeoGebra, in this activity the experimental group students were more involved than the control students (Zengin, Furkan and Kutluca, 2012). The problem-solving problems given to the control group were not as many as those given to the experimental group. In the experimental group, problem-solving can be done independently in various forms according to the desired form with the help of GeoGebra. Thus, the experimental group can solve the problem of the derivative function of two variables more than the control group. So that students are more involved in learning mathematics that is integrated with GeoGebra (Latifi, Hattaf and Achtaich, 2021). GeoGebra-assisted worksheets on the derivative of functions of two variables which are given to students at the beginning of learning, really help students to explore features related to the topic. Thus, using GeoGebra accelerates the students' understanding on the topic discussed (Yerizon, Fatimah and Tasman, 2021). The achievements of the experimental group were generally more prominent than the control group. This shows that mathematics learning that is integrated with GeoGebra can foster student interest and achievement, therefore it is very important to integrate the teaching and learning process with media that can satisfy students' interests (Wassie and Zergaw, 2019).

4. CONCLUSION

This study provides an illustration that learning the derivative of a function of two variables that are integrated with GeoGebra is very helpful in understanding the material. With this tool, students freely explore the solution to the problem of the derivative of a given two-variable function by changing the form and observing it. So that in this lesson students can try themselves repeatedly with the help of GeoGebra. This article is useful for math lecturers to improve their understanding of mathematics with the help of GeoGebra.

CONFLICT OF INTEREST

There are no conflicts of interest declared by the authors.

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