ABSTRACT
Harmful Einstellung effect creates a serious source of aversion for students in Mathematics. It may not be clear whether students fail Mathematics due to lack of knowledge or due to aversion arising from Einstellung effect. This study therefore investigated students’ susceptibility to harmful Einstellung effect in Mathematics instruction using logistic-empirical model approach. A cross-section of senior secondary school (SS 2) students of Alvana Model Secondary School Owerri, Imo State numbering 93 were continuously observed (using direct observational technique with observation schedule) and surveyed to decipher their susceptibility to harmful Einstellung effect. The result indicates increasing susceptibility to harmful Einstellung effect. That is, a high prevalence of Einstellung phenomenon was observed among students. It is important to note that this has consequently resulted in the increase of aversive behaviour in Mathematics learning. It was recommended among others that teachers should inform the students about harmful Einstellung effect during teaching and encourage them to think critically about any mathematical problem before finding solutions. Also, teachers should adopt some approaches when teaching Mathematics, such as, delayed formalization approach and target task approach.

1. INTRODUCTION.

One does not need a fortune teller to tell him or her the importance of Mathematics to an individual and his society. Mathematics has been highly rated among other subjects and for that reason, it has been described as the queen of all sciences and servant to all discipline. Without strong foundation in early Mathematics, students are not prepared to enroll in more advance Mathematics at university level (Obasi, 2014). It is unfortunate that students who regard Mathematics as hard are compelled to study one form of Mathematics or other. This is because Mathematics pervades all forms of learning. The acquisition of mathematical skills is mandatory for proper intellectual development (Harbor-Peters, 2001). It is therefore, not erroneous for Mathematics learning to be made one of the basic requirements for several courses or programmes. It is disheartening that the general performance of students in Mathematics has been observed to be poor (Obasi, Ezekwe, Mba, D & Ugo, 2016). There are actually quite a wide range of factors that could lead to difficulty in learning Mathematics. Some emanate from aversive aspect of school Mathematics. Sources of aversions consist of some of the teaching/learning processes, which may lead to failure and thus produce aversion. It should be noted that aversions are not deliberately created but come up as a result of activities that may create failure, fear which may lead to resentment against Mathematics (Harbor-Peters, 2001). These include harmful Einstellung effect, among others. However, this paper is on harmful Einstellung effect.

Einstellung is the development of a mechanized state of mind often called a problem-solving set (Dronek & Blessing, 2006). Einstellung refers to a person's predisposition to solve a given problem in a specific manner even though better or more appropriate methods of solving the problem exist. The Einstellung effect is the negative effect of previous experience when solving new problems. Harbor-Peters (2001) defined harmful Einstellung effect as the tendency to adhere to a previously practiced method of solving problem even when such method no longer offers the most direct, efficient and/or correct procedure for solving the problem. The Einstellung effect has been tested experimentally in many different contexts. The most famous example (which led to Luchins' coining of the term) is the Luchins' water jar experiment, in which students were asked to solve a series of water jar problems. After solving many problems which had the same solution, students applied the same solution to later problems even though a simpler solution existed (Luchins, 1942).

The Einstellung effect occurs when a person is presented with a problem or situation that is similar to problems they have worked through in the past. If the solution (or appropriate behaviour) to the problem/situation has been the same in each past experience, the person will likely provide that same response, without giving the problem too much thought, even though a more appropriate response might be available. Essentially, the Einstellung effect is one of the human brain's ways of finding an appropriate solution/behaviour as efficiently as possible. Note, however, that...
although finding the solution is efficient, the solution found might not necessarily be the most appropriate solution.

The Einstellung effect can be supported by theories of inductive reasoning. In a nutshell, inductive reasoning is the act of inferring a rule based on a finite number of instances. Most experiments on human inductive reasoning involve showing students a card with an object (or multiple objects, or letters, etc.) on it. The objects can vary in number, shape, size, color, etc. and the students’ job is to answer (initially by guessing) “yes” or “no” as to whether or not the card is a positive instance of the rule (which must be inferred by the students). Over time the students do tend to learn the rule, but the question is how? Kendler and Kendler (cited in Dronek & Blessing, 2006) proposed that older children and adults tend to exhibit noncontinuity theory; that is, the students tend to pick a reasonable rule and assume it to be true until it proves false.

Regarding Einstellung effect, one can view noncontinuity theory as a way of explaining the tendency to maintain a specific behaviour until it fails to work. In the water-jar problem, students generated a specific rule because it seemed to work in all situations; when they were given problems for which the same solution worked, but a better solution was possible, they still gave their tried and true response. Where theories of inductive reasoning tend to diverge from the idea of Einstellung effect is when analyzing the fact that, even after an instance where the Einstellung rule failed to work, many students reverted to the old solution when later presented with a problem for which it did work (again, this problem also had a better solution). One way to explain this observation is that in actuality students know (consciously) that the same solution might not always work, yet since they were presented with so many instances where it did work, they still tend to test that solution before any other (and so if it works, it will be the first solution found).

Harbor-peters (2001) found that harmful Einstellung effect exists very much in some mathematical problems and creates a serious source of aversion for both low and high achievers in Mathematics. The following examples will illustrate harmful Einstellung effect. This is based on the report of Harbor-Peters (2001), who presented the problem to a group of participants in a conference.

Example on Algebra

According to Harbor-Peters (2001), only one mathematician was able to solve the problem. Most of the participants tried to solve the problem using the Pythagorean Theorem. This approach could not produce any solution. This is because they wanted to find the hypotenuse of a right-angled triangle. Clearly, this is an application of a previously practiced method of a solution process (a case of harmful Einstellung). However, this method did not produce a solution to the given problem. This problem could be easily solved thus: Since the figure POST is a rectangle, its diagonals are congruent. OT is a diagonal and is also the radius of the circle O. Since OT = 7, it follows that PS = 7, and AO = OT = PS = 7.

Example on Geometry

Considering the diagram below, AB and CD are diameters of circle O and AB is perpendicular to CD. Figure POST is a rectangle, AP = 3 and PO = 4. Find the length of PS.

![Diagram](image.png)

**Fig. 1. Example on Geometry**

Thus: Since the figure POST is a rectangle, its diagonals are congruent. OT is a diagonal and is also the radius of the circle O. Since OT = 7, it follows that PS = 7, and AO = OT = PS = 7.

Example on Algebra

4 cups of beans and 7 cups of rice cost N153.00, 6 cups of beans and 3 cups of rice cost N117.00. What is the cost of 10 cups of beans and 10 cups of rice?

Most students would solve this problem by using these sets of equations:

\[4b + 7r = 153\]  \hspace{1cm} (1)

\[6b + 3r = 117\]  \hspace{1cm} (2)

where \(b\) = beans and \(r\) = rice

After laborious working, a student will arrive at \(b = 12\) and \(r = 15\). She/he will then proceed to substitute: \(10b + 10r = 10(12) + 10(15)\) to obtain 270. Actually, the expression \(10b + 10r = 270\). But because of the long habit of solving problems using set methods, it prevents the student from trying any other method. This is a case of harmful Einstellung (Harbor-Peters, 2001).

It may be necessary to determine whether the students fail a question because of lack of mastery of a concept or merely by being trapped by an Einstellung developed from responding to previous mathematical items or other sources. Harmful Einstellung do create aversion for Mathematics learning since it leads to failure. Susceptibility to harmful Einstellung was observed not only on students of low or average mathematical abilities, but also on students of high mathematical abilities (Harbor-Peters, 2001). This goes to show the prevalence of harmful Einstellung effect in Mathematics among students irrespective of their ability levels.

Logistic Mathematical Model

This paper resorted to the mathematical model due to Pierre-Francois Verhulst. This is a model of human population
whose rate of change is given as:
\[
\frac{dP}{dt} = P(\alpha - \delta P)
\]

where \(\alpha\) and \(\delta\) are positive constants and \(P\) is the population with time. The model above is called logistic equation, its solution is called logistic function and the graph is called the logistic curve. Logistic model has proved quite accurate in predicting growth pattern (Ugwu & Ikpegbu, 2000).

**Method of Solving Logistic Model**

Separation of variables is used to solve the logistic equation of the model. Thus, the required solution is given as:
\[
P(t) = \frac{\alpha P_0}{\delta P_0 + (\alpha - \delta P_0)e^{-\alpha t}}
\]

### 2. METHODS

The study adopted a developmental survey design of cross-sectional approach, where senior secondary school (SS2) students of Alvana Model Secondary School Owerri, Imo State numbering 93 were continuously observed and on the fifth day of ascertaining those who are susceptible to harmful Einstellung, 62 students were found to have developed Einstellung phenomenon. The observations were carried out for some days and the data were generated. The fifth day observation was used to predict the rest and comparison was done between the computed and observed data. The students were observed using direct observational technique (with observation schedule) within the space of 12 days.

### 3. RESULTS AND DISCUSSION

Considering the following values \(\alpha = 93k\), \(P_0 = 1\) and \(\delta = k\), thus from the logistic function, we have the following logistic-empirical model based on data fitting:
\[
P(t) = \frac{93}{1 + 92e^{-1.0429t}}
\]

Based on the above logistic-empirical model, the other computed values and observed values of students who are susceptible to harmful Einstellung in the classroom are shown in Table 1.

**Table 1. Computed and Observed Values**

<table>
<thead>
<tr>
<th>(t) (days)</th>
<th>(P) (Observed)</th>
<th>(P) (Computed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>62</td>
<td>62</td>
</tr>
<tr>
<td>6</td>
<td>73</td>
<td>79</td>
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<td>7</td>
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<td>10</td>
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<td>11</td>
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<td>93</td>
</tr>
<tr>
<td>12</td>
<td>92</td>
<td>93</td>
</tr>
</tbody>
</table>

*Source: Researchers’ Field Survey, 2017*

Table 1 shows the computed and observed values based on the survey. The table suggests high susceptibility to harmful Einstellung effect. The logistic curve illustration is shown below.

**Logistic Curve Based on Observed and Computed Data**

![Logistic Curve](image-url)

*Fig. 1. Curve of \(P\) (observed/computed) against \(t\) (days)*
The logistic curves in figure 1 clearly display the information in Table 1 that suggests high susceptibility to harmful Einstellung effect.

The results from Table 1 and the logistic curves in Figure 1 showed that the observed values and the computed or predicted values almost correspond (converges), indicating increasing susceptibility to harmful Einstellung effect among the students. That is, a high prevalence of Einstellung phenomenon. It is important to note that this has consequently resulted in the increase of aversive behaviour in Mathematics learning. This implies that harmful Einstellung do create aversion for Mathematics learning since it leads to failure. This result agrees with Harbor-Peters (2001) who reported that susceptibility to harmful Einstellung was observed not only on students of low or average mathematical abilities, but also on students of high mathematical abilities. This by implication goes to show the prevalence of harmful Einstellung effect in Mathematics among students irrespective of their ability levels.

4. CONCLUSION

This result makes it clear that students may have been failing Mathematics not because of lack of knowledge/mastery but due to aversion arising from harmful Einstellung effect. Therefore, teachers should inform the students about harmful Einstellung effect during teaching and also encourage them to think critically about any mathematical problem. More so, teachers should adopt some approaches when teaching Mathematics, such as, delayed formalization approach and target task approach.

Acknowledgment

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REFERENCES


