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Research Article

Problem-based scaffolding for prospective mathematics teachers in graph theory course

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ABSTRACT
This study aimed to give a scaffolding process to improve the preservice teachers' ability in graph theory class. The present study implemented scaffolding techniques by administering graph theory questions. Four scaffoldings are used: (1) questioning, (2) prompting, (3) cueing, and (4) direct explanation. The choice of this problem can be observed through students' comprehension of graph theory. This research indicates that problem-based scaffolding can benefit students with challenges understanding graph theory. The students exhibit enthusiasm as they explore the interconnection between road systems, predatory and isomer, employing graph theory.

Keywords: Problem-Based Learning; Scaffolding; Prospective Teachers; Graph Theory; Mathematics Learning

1. INTRODUCTION
Graph theory holds significant importance within the realm of mathematics (Sporns, 2018; Farahani, Karwowski and Lighthall, 2019; Medová et al., 2019; Uyangör, 2019; Wulandari and Damayanti, 2019; Wahyuningsih, Satyananda and Qohar, 2020). Graph Theory finds several applications in various domains, including but not limited to computing and transportation (Hasmawati, 2015; Shao et al., 2020; Wang and Tang, 2021; Chen et al., 2022). Hence, the inclusion of Graph Theory in the curriculum is obligatory for students pursuing a mathematics degree (Marion, 1991; Al-jawadi and Al-shumam, 2020). In practical application within the domain, numerous students encounter difficulties acquiring knowledge in the subject matter. A perception exists among many students that Graph Theory is a relatively recent concept (ธาดา, ราช and ประสิทธิ์, 2013; Suwanti, 2016; Arosio, Martina and Figueiredo, 2020; Aziz, 2021; Bahrudin and Isnani, 2022). The findings from the Graph Theory class at Wisnuwardhana University indicate that many students had challenges while attempting to create graph representations. Hence, providing aid or scaffolding becomes imperative to address these challenges.

Scaffolding refers to the instructional approach wherein a knowledgeable someone assists students in resolving an issue, accomplishing a task, or attaining a goal that necessitates a degree of comprehension that can only be attained through guidance and assistance (Anghileri, 2006; Fisher and Frey, 2010; Chairani, 2015; Fajrjani, Naswir and Harizon, 2021; Kusmaryono, 2021; Chun and Cennamo, 2022; Pramerta, 2022). The employment of scaffolding techniques has been found to have a beneficial effect on facilitating the learning process (Wulandari and Damayanti, 2019; Chen and Tseng, 2021). According to Vygotsky (Pramerta, 2022; Wulandari & Damayanti, 2018), the implementation of scaffolding by teachers can provide support for problem-solving in children with learning impairments. Scaffolding refers to providing assistance or guidance in problem-solving endeavors, typically offered by an individual possessing expertise in the subject matter (Safitri et al., 2023). The reason behind enhancing students' comprehension lies in the provision of robust cognitive scaffolding (Lönngren, Adawi and Svanström, 2019; Ahmed Abdel-Al Ibrahim et al., 2023). There exist numerous ways that can be employed in the facilitation of scaffolding. Various scaffolding methods encompass game-based approaches, peer tutoring strategies, problem-solving techniques, and further methodologies. According to the findings of Ernawati et al.’s study, the implementation of scaffolding through problem-based learning significantly impacts the development of students' creative thinking abilities (Cahyono et al., 2021; Ernawati et al., 2023). According to Fisher (2010), instructors engage in scaffolding that involves four distinct components while assisting pupils: (1) Employing questioning to interrogative techniques to assess comprehension; (2) Employing prompts to enhance students' cognitive processes; (3) Employing cues to redirect students' attention towards examining more specialized information and identifying and addressing faults or...
incomplete comprehension; (4). Providing explain clarification for pupils who lack the understanding to accomplish the assigned assignment (Fisher and Frey, 2010; Qamar and Riyadi, 2016; Haryati et al., 2020; Tiaradipa et al., 2020). Problem-based learning (PBL) is an instructional approach that places the student at the center of the learning process to promote active engagement and enhance the meaningfulness of the learning experience (Sudarsana et al., 2019; Asrial et al., 2021; et al., 2022; Wulandari, Khasanah and Octavianti, 2022; Rafiq, Triyono and Djatmiko, 2023; Ssemugenyi, 2023). Problem-based learning has the potential to generate enthusiasm among students for the study of Graph Theory (Wulandari, Khasanah and Octavianti, 2022).

Based on the preliminary observation, it has been noted that certain pupils encounter challenges when attempting to ascertain the quantity of edges within a graph. The subsequent passage illustrates the discoveries made by scholars in the challenges encountered by students in comprehending the principles of graph theory.

Thus, the researchers employed scaffolding techniques in this study to address the challenges faced by students, namely by introducing authentic graph theory issues. The aim was to facilitate the process of overcoming these difficulties.

2. RESEARCH METHOD
This type of study is Qualitative research. The study was conducted on third-year students at Wisnuwardhana University Malang. The Graph Theory class was held once every week. At the beginning of the Graph Theory course, a brief review of the set. From the learning process, students have difficulty with concepts in Graph representation, which will be given scaffolding. The participants in this study were lecturers teaching Graph Theory courses and students in one class/offering. One class consists of heterogeneous male and female students in terms of mathematical ability. Data were collected through practice problems consisting of student work, interviews with lecturers, and field notes (Bassey, 2000). In this study, the scaffolding given to students is based on contextual problems that have been designed in such a way as to assist students in solving a mathematical problem on Graph Theory material, especially the concept of Graph representation. This study discusses the scaffolding process of 4 different problems: the road system problem, food chain predator behavior, chemical isomers, and graph isomorphism.

3. RESULTS AND DISCUSSION
Table 1. Forms of Questioning Scaffolding (Road System Problem)
According to the data presented in Table 1, the subject can provide answers. However, it is recommended to employ scaffolding techniques such as asking to ascertain the subject’s comprehension of the answer. This will allow for a thorough assessment of the subject’s understanding of the question and their response.
Researchers draw the graph representing the road system in the figure beside, and determine the number of vertices, the number of edges, and the degrees of each vertex. Can you draw it? (because the subject did not draw the graph, only mentioned the number of vertices, edges, and degree of each vertex).

Subject: Yes, ma'am. (subject starts to draw the graph)

Researcher: Try to label the graph according to the known image.

Subject: Here you go, ma'am, labeling the graph.

Researcher: Correct. Now, explain how to determine the number of vertices, edges, and degree of each point.

Subject: There are 5 vertices, ma'am, 7 sides, and the degree of vertices P=2, Q=4, R=2, T=3, S=3.

Researcher: What are the vertices? What is the degree of the vertex?

Subject: The vertices are P, Q, R, S, T. If the vertex degree is obtained from the number of edges directly connected to the vertex. P=2 (Q, S); Q=4 (P, S, T, R); R=2 (Q, T); T=3 (R, Q, S); S=3 (P, Q, T)

**Table 2. Forms of Prompting Scaffolding (Predatory digraph problem)**

According to the data presented in Table 2, it can be observed that the participant exhibited a delay in submitting his response, prompting the provision of questioning scaffolding. To provide scaffolding in the form of inquiry, it is seen that the subject’s response still contains faults. Consequently, when scaffolding is provided through prompting, the subject becomes aware of the location of their mistake.

Researcher: Snakes are known to consume frogs, whereas birds are known to prey on spiders. It is worth noting that both birds and spiders exhibit a dietary preference for insects. Additionally, frogs are observed to consume snails, spiders, and insects as part of their feeding behavior. Please create a digraph illustrating the predatory behavior described.

Subject: Bu Like this Mum

Researcher: Are you sure about the digraph?

Subject: Long silence. Sure Mum

Researcher: Explain the predatory behavior of the problem presented.

Subject: Snakes are known to consume frogs, while birds are known to prey on spiders. It is worth noting that birds and spiders have a dietary preference for insects. Additionally, frogs exhibit a feeding behavior that includes the consumption of snails, spiders, and insects. We have been tasked with creating a digraph depicting the predatory behavior.

Researcher: What do the vertices represent? And what do the directed edges represent?

Subject: The conclusion is that animals in problems and directed sides eat. Right ma'am?

Researcher: The vertices are the animals in the problem, and the directed side is eating. Right, mum?

Subject: (Observing) Snakes eat frogs, right Frogs eat spiders, insects, and snails. Correct Spiders eat insects, right Insect eats bird. Wrong The bird eats spider, right Something is wrong, Mum; it should be an insect, not an insect-eating bird. So, the direction of my directed side needs to be corrected.

Researcher: Right, now you understand. Then what's the keyword?

Subject: As you asked earlier, we must know what the vertex represents and the directed side.

Researcher: Correct.
Table 3. Form of Cueing Scaffolding (Chemical Bonding Problems)

According to the data presented in Table 3, the participant has employed reasoning to address the problem. However, uncertainty is still expressed by the statement, "Still need to think of it, Mum" Therefore, it is necessary to provide scaffolding in the form of cueing, as illustrated in Table 3.

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Figure beside depicts the chemical structures of methane (\text{CH}_4) and propane (\text{C}_3\text{H}_8). (i) When considering these diagrams as graphs, it is crucial to analyze the characteristics of the vertices that represent carbon atoms (C) and hydrogen atoms (H). There exist two distinct chemical compounds denoted by the formula (\text{C}<em>4\text{H}</em>{10}). Please illustrate the molecular structures by plotting the related graphs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject</td>
<td>Here's the thing, Mum</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Is there another form with the same molecular formula (\text{C}<em>4\text{H}</em>{10})?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject</td>
<td>Long silence</td>
</tr>
<tr>
<td>Researcher</td>
<td>How is it? Are there any other forms?</td>
</tr>
<tr>
<td>Subject</td>
<td>Confused, Mum, that is it.</td>
</tr>
<tr>
<td>Researcher</td>
<td>Notice what the vertices and sides are?</td>
</tr>
<tr>
<td>Subject</td>
<td>The nodes are Hydrogen atoms and carbon atoms.</td>
</tr>
<tr>
<td>Researcher</td>
<td>Try to observe that carbon atoms have what degree? And hydrogen atoms have what degree?</td>
</tr>
<tr>
<td>Subject</td>
<td>(\text{observe}) Carbon atoms have degree 4 and hydrogen atoms have degree 1.</td>
</tr>
<tr>
<td>Researcher</td>
<td>Right now, try to make a graph of another molecular arrangement of (\text{C}<em>4\text{H}</em>{10}).</td>
</tr>
<tr>
<td>Subject</td>
<td>One second, mum (\text{thinking})</td>
</tr>
<tr>
<td>Researcher</td>
<td>The key is that the Carbon atom has degree 4, and the hydrogen atom has degree 1.</td>
</tr>
<tr>
<td>Subject</td>
<td>Still need to think of it, Mum.</td>
</tr>
<tr>
<td>Researcher</td>
<td>Do Carbon atoms always have to be straight chains? Try to make the Carbon atoms branched. Can it be?</td>
</tr>
<tr>
<td>Subject</td>
<td>I'll try it first mum.</td>
</tr>
<tr>
<td>Researcher</td>
<td>ok, try to make it first.</td>
</tr>
<tr>
<td>Subject</td>
<td>Tell you what Mum.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Researcher</th>
<th>Yes, that's right. Do you understand? What is the keyword, then?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject</td>
<td>We refer to the degree of Carbon atoms and Hydrogen atoms. Mum</td>
</tr>
<tr>
<td>Researcher</td>
<td>Yes, that's right, so that we can create different graphs from the arrangement of molecules with the same molecular formula. These graphs are called isomers in chemistry.</td>
</tr>
</tbody>
</table>

Table 4. Form of Explaining Scaffolding (Graph degree sequence problem)

According to the data presented in Table 4, the scaffolding approach that exhibits the highest level of support is the explained scaffolding form. This form is preceded by the questioning, prompting, and cueing scaffolding forms. This phenomenon may occur when the subject's comprehension remains uncertain, facilitating the acceptance of the process through explanatory scaffolding.
<table>
<thead>
<tr>
<th>Researcher</th>
<th>Determine the statement's veracity: &quot;Any two graphs with identical degree sequences are isomorphic.&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject</td>
<td>True, Mum.</td>
</tr>
<tr>
<td>Researcher</td>
<td>Are you sure?</td>
</tr>
<tr>
<td>Subject</td>
<td>A long silence, sure, mum. For example, the problem with the isomers earlier, mum.</td>
</tr>
</tbody>
</table>

Researcher: Can you explain what a degree sequence is?

Subject: The degree sequence of a graph consists of vertex degrees written in increasing order, with repetition where necessary. (Pointing) This is the same degree sequence 1,1,1,1,1,1,1,4,4,4,4,4,4.

Researcher: Try to explain what the problem is.

Subject: Determine the statement's veracity: "Any two graphs with the same degree sequence are isomorphic."

Researcher: Are you sure this statement is true?

Subject: (thinking) sure Mum.

Researcher: Are you sure there are no two graphs that are not isomorphic but have the same degree sequence?

Subject: Just a moment, mum (thinks for a while). Sure mum

Researcher: How?

Subject: Still confused mum

Researcher: What are the characteristics of two graphs that are isomorphic to each other?

Subject: What is it? One moment, ma’am. The characteristics: 1). The number of vertices in $G_1$ equals the number of vertices in $G_2$. 2). The number of edges in $G_1$ is equal to the number of edges in $G_2$. 3). The degree of each vertex corresponding to each other in both graphs is the same.

Researcher: Correct, that is the characteristic that two graphs are isomorphic. Now, find if two graphs have the same degree sequence but are not isomorphic.

Subject: (Thinking for a long time) still haven’t thought of it, Mum

Researcher: Try to find the statement "Any two graphs with the same degree sequence are isomorphic" can be interpreted to mean that all graphs with the same degree sequence are isomorphic. If you find even one example of two graphs with the same degree sequence that are not isomorphic, then the statement is false. Do you still believe that the statement is true?

Subject: So I don't know if Ma’am is right. But if it’s wrong, I have yet to find an example.

Researcher: Try to find two more graphs that do not fulfill the isomorphic feature but have the same degree sequence.

Subject: (thinking for a long time) I haven’t found it yet, Mum

Researcher:

![Graphs $G_1$ and $G_2$](image)

Consider the graphs $G_1$ and $G_2$. What is their degree sequence?

Subject: $G_1$ the degree sequence is (1,1,2,2,2)  
$G_2$ the degree sequence is (1,1,2,2,2)  
the degree sequence of $G_1$ dan $G_2$ are the same Ma’am.
Researcher: Are graphs $G_1$ and $G_2$ isomorphic?

Subject: No Mum

Researcher: Which characteristics are not fulfilled?

Subject: 1). The number of vertices in $G_1$ equals the number of vertices in $G_2$, which is 5. 2). The number of edges in $G_1$ is equal to that in $G_2$, which is 4. 3). The degree of each vertex corresponding to each other in the two graphs is different, Ma’am.

Researcher: Show me which one is different.

Subject: In graph $G_1$, the vertex of degree 1 is directly connected to the vertex of degree 2, while in graph $G_2$, the vertex of degree 1 is directly connected to the vertex of degree 1.

Researcher: Yes, it is. So now, how is the statement true or not?

Subject: False, Ma’am

Researcher: What makes the statement false?

Subject: There is an example that undermines this statement, ma’am.

Researcher: That’s right. Do you understand now? So, we must carefully determine whether a statement is true or false. We have to check it.

Subject: Yes, ma’am, I didn’t think of that before. Some do not fulfill.

4. CONCLUSION

Based on the results and discussion, problem-based scaffolding is very helpful in learning. Contextual problems help in bridging the abstractness of graph theory. The scaffolding done in this study varied from questioning, prompting, cueing, and explaining. The choice depends on the response of the students.

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CONFLICT OF INTEREST

There are no conflicts of interest declared by the authors.

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